Do Vertical Mergers Facilitate Upstream Collusion?

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We investigate the impact of vertical mergers on upstream firms’ ability to collude when selling to downstream firms in a repeated game. We show that vertical mergers give rise to an outlets effect: the deviation profits of cheating unintegrated firms are reduced as these firms can no longer profitably sell to the downstream affiliates of their integrated rivals. Vertical mergers also result in an opposing punishment effect: integrated firms typically make more profit in the punishment phase than unintegrated upstream firms. The net result of these effects in an unintegrated industry is to facilitate upstream collusion. We provide conditions under which further vertical integration also facilitates collusion. (JEL D43, G34, L12, L13)

Many famous cases of collusion have involved intermediate goods industries. Further, a significant fraction of those cases involved industries where one or more firms were vertically integrated.1 Yet existing theories of collusion deal only with collusion between firms selling to consumers (or atomless buyers). In this paper, we provide the first examination of the often more relevant case where colluding firms sell to downstream firms which are strategic buyers with interdependent demands. Our particular focus is on the effect of vertical integration on the possibility of collusion in such markets. Why is vertical integration such a common feature of collusive industries? Does vertical integration facilitate upstream collusion, and if so, when should it be a concern for antitrust regulators?

The Chicago School antitrust revolution of the 1970s revealed serious flaws in earlier analyses of the effects of vertical mergers and restraints and showed that such restraints were often efficiency-enhancing. Nevertheless, it is now generally accepted that vertical mergers and restraints may be anticompetitive if they either raise their rivals’ costs or help firms commit to a lower output (higher price).2 But the literature underpinning these results has so far taken a steel cartels, as well as Margaret C. Levenstein’s (1997) description of the bromine cartel. Other examples of collusion involving some vertically integrated firms include railways (Robert H. Porter 1983) and timber-cutting (Laura H. Baldwin, Robert C. Marshall, and Jean-François Richard 1997). See also Kenneth Hendricks, Porter, and Guofo Tan (2000) on joint bidding for oil and gas tracts, and Frederic M. Scherer (1980) for a general discussion.

1 See for example Harry R. Tosdal’s (1917) description of vertical mergers in the early twentieth century German
strictly static view of the interaction between firms. In contrast, in this paper we investigate the impact of vertical mergers in a dynamic game of repeated interaction between upstream and downstream firms.

We consider an industry where, in each period, \( M \) upstream firms without capacity constraints produce a homogeneous intermediate good and make public two-part tariff offers to supply this good to \( N \) downstream firms. The downstream firms purchase the intermediate good and transform it into a homogeneous or differentiated final good, competing in either prices or quantities to supply consumers. This interaction is repeated over an infinite horizon. We focus our analysis on collusion between the upstream firms, which aim to implement the monopoly outcome at the industry level and then extract the rents from this upstream.\(^3\) We will say that a vertical merger facilitates collusion if it reduces the critical discount factor above which this monopoly outcome is sustainable using so-called trigger strategies: infinite reversion to the repeated play of the static equilibrium of the stage game following a deviation by one of the firms (e.g., James W. Friedman 1971).\(^4\)

Perhaps the most intuitive and important effect of vertical merger on collusion possibilities is the outlets effect. To understand this effect, consider first that the optimal way for an upstream firm to deviate from collusion is typically to undercut the fixed fees and wholesale prices of its rivals only marginally. This allows the deviant firm to steal all of its rivals’ business while downstream output remains close to monopoly levels, and hence the deviant firm’s profits are close to monopoly profit. Note that such a strategy is no longer feasible when one or more downstream firms are integrated. Integrated downstream firms will always prefer to buy from their upstream affiliate at marginal cost than to buy from a deviant firm at any price that gives the latter positive profits (essentially, because they would rather these profits go to their upstream affiliate than to another firm). Thus, integrated downstream firms can be relied upon to reject any offer that would be profitable for a deviating upstream firm, which can help to enforce the collusive agreement. An upstream firm cannot hope to attain almost the entire monopoly profit when deviating if one or more downstream firms are integrated with its rivals. We call this the outlets effect of vertical integration, since vertical integration by an upstream firm reduces the number of outlets through which its rivals can sell when deviating, generally reducing their profit from cheating and thus facilitating collusion.

Counteracting the outlets effect is the punishment effect. This is also quite intuitive and arises in our setup because downstream firms may earn positive profits in the noncooperative equilibrium of the model. (A deviation from collusion is assumed to lead to this noncooperative equilibrium in perpetuity.) If an upstream firm integrates with a downstream firm, these profits now become part of the profit of the merged entity. Thus, the merged entity can expect to make more profits in the noncooperative punishment phase than the upstream firm would make alone. Absent any changes in market share, however, the merged entity will make the same profit as a stand-alone upstream firm when monopoly profits are sustained by collusion upstream. So, for a given collusive market share, the merged entity suffers less than a stand-alone upstream firm from a switch from collusive to punishment phases, and is correspondingly more tempted to cheat on any collusive agreement. We call this the punishment effect of vertical integration since it arises because the noncooperative equilibrium is a less harsh punishment of a vertically merged firm than of a stand-alone upstream firm.

The plan of this article is as follows. We begin, in Section I, with a numerical example to illustrate the main intuitions behind our results. In Sections II and III, we set out and analyze our baseline model, where upstream

\(^3\) In dealing with the collusive effects of vertical merger, the US Non-Horizontal Merger Guidelines also focus on the impact on upstream collusion. For a discussion of how our theory relates to these guidelines, see Nocke and White (2003).

\(^4\) Note that our focus on monopoly outcomes is less restrictive here than it might at first appear because it turns out that the Bertrand-like structure of competition in the upstream market means that the ability to collude has an all-or-nothing feature when no firm is integrated: whenever it is possible to sustain an equilibrium with positive profits for upstream firms, it is possible to sustain an equilibrium in which all of the monopoly rents are extracted by upstream firms and shared between them; moreover, these monopoly equilibria are preferred by unintegrated firms to other collusive equilibria on the Pareto frontier.
and downstream firms set their prices or quantities simultaneously. In this game we identify the two counteracting effects of vertical mergers mentioned above. We show that, quite generally, the outlets effect outweighs the punishment effect so that the net effect of the first vertical merger is to facilitate collusion, according to the definition set out above. In addition, we show in Section IIIIE that vertical merger also facilitates coordination on equilibria with prices below monopoly levels when the latter are not sustainable; and that, in addition, it might plausibly lead to coordination on asymmetric equilibria which are socially even less desirable than the monopoly equilibrium. In Sections IIIF and IIG, respectively, we discuss the incentives for vertical merger and the effect of multiple vertical mergers in an industry. In Section IV, we change the timing of the baseline model to allow downstream firms to set their strategic variable after they have observed upstream offers, and we show that two other effects—the reaction effect and the lack-of-commitment effect—arise, both of which further facilitate collusion relative to the baseline model. Section V concludes.

I. A Numerical Example

To understand the various effects arising from vertical merger, consider the following numerical example, summarized in Table 1. Suppose that there are five upstream firms and five downstream firms operating in an industry, and that the total profit of a vertically integrated monopolist in this industry structure would be 100 (with each downstream firm selling one-fifth of the monopoly quantity). In the absence of vertical integration, the upstream firms can each earn 20 (= 100/5) every period if they successfully collude. (To do so, they must set wholesale prices in such a way as to induce downstream firms to charge monopoly prices, and then extract the resulting profits through fixed fees). If, on the other hand, the upstream firms do not collude, then suppose that they will earn zero profits in the noncooperative equilibrium, whereas the five downstream firms will earn noncooperative profits of 10 each. By cheating and slightly undercutting the collusive offers, an upstream firm can capture the entire monopoly profit of 100 for one period, versus 20 now and forever from colluding. The incentive to cheat, as given by the ratio between the one-off gain from cheating and the foregone profit from colluding, is thus equal to (100 − 20)/20 = 4 (or, equivalently, the critical discount factor above which upstream collusion is sustainable is 4/(1 + 4) = 0.8).\(^5\)

If one upstream firm now integrates with a downstream firm, the merged entity can earn 30 (rather than 0) if collusion breaks down and the firms return to noncooperative play. We call this increase in the integrated firm’s incentive to deviate the punishment effect. The punishment effect makes collusion more difficult to sustain, and captures the intuitive idea that it is harder to punish a vertically integrated firm than an un integrated upstream firm. For vertical merger to facilitate upstream collusion, the integrated firm must therefore be given a larger share of the collusive pie. To fix ideas, suppose the integrated firm earns a per-period profit of 30 when colluding (20 from the sales of its own downstream affiliate and 10 from sales to one or more of the four unintegrated downstream firms), while

\(^5\) Collusion can be sustained if and only if the one-off gain from cheating is less than or equal to \(\delta/(1 - \delta)\) times the foregone profit from colluding, where \(\delta\) denotes the discount factor.
the remaining monopoly profit of 70 is equally shared among the four unintegrated upstream firms. Consider, now, the incentive to cheat of the four remaining unintegrated upstream firms. When colluding, each earns \( 70/4 = 17.5 \) per period. But when cheating, an unintegrated upstream firm cannot profitably sell to the integrated downstream firm as the latter internalizes the loss in profit to its upstream affiliate caused by switching suppliers. An unintegrated upstream firm can thus obtain at most only 80 when it cheats (rather than 100 in the absence of vertical integration). This reduction in an unintegrated upstream firm’s deviation profit is what we call the outlets effect of vertical integration. In fact, the outlets effect outweighs the countervailing punishment effect: the vertical merger reduces the incentives to cheat for all upstream firms. The ratio between the one-off gain from cheating and the foregone future profit is now only \((80 - 17.5)/17.5 \approx 3.57\) for an unintegrated upstream firm and \((100 - 30)/(30 - 10) = 3.5\) for the integrated firm. This implies that the vertical merger has reduced the critical discount factor above which upstream collusion is sustainable.

Now suppose that downstream firms set their prices or quantities after the upstream offers have been made and accepted or rejected. (Above, we implicitly assumed that downstream prices are set at the same time as upstream offers are made. This implied that in the period when one of the upstream firms deviates from collusion, each one of the five downstream firms still sells one-fifth of the monopoly quantity.) When no firm is vertically integrated, a deviant upstream firm can still obtain the monopoly profit of 100 when deviating. The deviating upstream firm will optimally offer contracts that induce each downstream firm to continue to charge the monopoly price, and extract the entire monopoly profit, just as in the simultaneous case. But things are different when one firm is vertically integrated. In this case, an integrated downstream firm will optimally react to a deviation by an unintegrated upstream firm by immediately adopting noncooperative pricing: given that collusion will break down in the next period anyway, it is in the integrated downstream firm’s best interest to charge the myopic best-response price to the prices charged by the other downstream firms. Consequently, an unintegrated firm’s deviation profit is no longer 80 but only, say, 70. This reduction in deviation profit is the reaction effect of vertical merger. For the same reason, when the integrated upstream firm cheats in the upstream market, unintegrated downstream firms anticipate that it will follow this deviation by reducing its own downstream price. (This is a myopic best response since the integrated firm’s variable profit per unit of input sold to an unintegrated downstream firm is less than that sold through its own downstream affiliate.) This expected downstream price cut makes the unintegrated downstream firms willing to pay the integrated firm less for the input when it cheats, implying that the integrated firm can gain only, say, 90, rather than 100 from cheating. We call this the lack-of-commitment effect of vertical merger as it arises from the integrated firm’s inability to commit to the collusive downstream price. Both the reaction effect and the lack-of-commitment effect provide further reasons to believe that vertical integration facilitates collusion.

II. The Baseline Model

We consider a vertically related industry with \( M \geq 2 \) identical upstream firms, \( U_1, U_2, \ldots, U_M \), and \( N \geq 2 \) symmetric downstream firms (or retailers), \( D_1, D_2, \ldots, D_N \). The upstream firms produce a homogeneous intermediate good at constant marginal cost \( c \), which for simplicity we set equal to 0, and sell this good to the downstream firms. The downstream firms transform the intermediate good into a final good on a one-to-one basis at zero marginal cost of production, and sell it to consumers. Consumers view the final good as either homogeneous or symmetrically differentiated (by downstream firm).

The \( M \) upstream firms make simultaneous and public take-it-or-leave-it two-part tariff offers to the downstream firms. \( U_j’s \) offer to \( D_j \) takes the form \((w_{ij}, F_{ij})\), where \( w_{ij} \) is the marginal wholesale price and \( F_{ij} \) is the fixed fee. The fixed fee \( F_{ij} \) has to be paid when the offer is accepted, while the wholesale price \( w_{ij} \) has to be paid for each unit that is subsequently ordered. In the retail market, the \( N \) downstream firms compete either in prices or quantities. That is, \( D_j \) sets a retail price \( p_j \) (under price competition) or quantity \( q_j \) (under quantity competition).

Time is discrete and indexed by \( t \). Each period, an identical set of consumers comes to
the downstream market to buy the final good; we set out the timing in each period in more detail below. Demand for downstream firm $D_j$’s final good is given by $Q(p_j; p_{-j})$, where $p_j$ is the price of $D_j$’s final good, and $p_{-j}$ the vector of prices charged by $D_j$’s downstream rivals. Downstream firm $D_j$’s inverse demand is denoted $P(q_j; q_{-j})$, where $q_j$ is $D_j$’s output, and $q_{-j}$ a vector of outputs by the $N - 1$ downstream rivals. We impose standard assumptions on the symmetric demand system. We assume that joint profits are maximized under a symmetric price vector $(p^M, \ldots, p^M)$ (or quantity vector $(q^M, \ldots, q^M)$). The associated joint-profit maximizing industry output and profit are denoted by $Q^M$ and $\Pi^M$, respectively. The following linear demand system, frequently used in oligopoly models, satisfies these assumptions, and we will later use it to illustrate some of our results.

EXAMPLE 1 (Linear Demand): There is a unit mass of identical consumers with utility function

$$U(x; H) = \sum_{j=1}^N (x_j - x_j^0)^2 - 2\sigma \sum_{j=1}^N \sum_{k=1}^N x_j x_k + H,$$

where $x_j$ is consumption of downstream firm $D_j$’s product, and $H$ is consumption of the Hicksian composite commodity. The parameter $\sigma \in (0,1]$ measures the degree of substitutability between products: products become perfect substitutes as $\sigma \to 1$, and independent as $\sigma \to 0$.

The timing in each period is as follows:

• Pricing stage: Upstream firms $U_1, \ldots, U_M$ simultaneously make public contract offers to the downstream firms. At the same time, downstream firms $D_1, \ldots, D_N$ simultaneously commit to prices (or quantities) in the retail market.

• Sunspot stage: A public random variable $\theta$, uniformly distributed on $[0,1]$, is realized.

• Acceptance stage: Downstream firms $D_1, \ldots, D_N$ simultaneously decide which contract(s) to accept. If they decide to accept a contract, the relevant fixed fee is paid to the upstream firm.

• Consumption stage: Consumers decide which final goods to purchase. Downstream firms then order the quantities demanded by consumers from the upstream firms at the relevant wholesale prices, and firms’ revenues are realized.

The game is one of perfect monitoring: all past actions become common knowledge at the end of each stage. Each firm aims to maximize the discounted sum of its future profits over an infinite horizon, using the common discount factor $\delta \in (0,1)$. Vertically integrated firms are assumed to maximize their joint profits. This implies that the vertically integrated downstream firm’s true wholesale price is the marginal cost of its upstream affiliate, $c = 0$ (Giacomo Bonanno and John Vickers 1988). To focus on the potential collusive effects of vertical integration, we assume that a vertical merger does not affect costs or technology.

Since we are interested in tacit collusion between upstream firms, we will focus mainly on collusive equilibria that allow upstream firms to jointly extract all of the monopoly rents. For simplicity, we assume that upstream firms sustain collusion through infinite “Nash reversion”: any deviation by an upstream firm or an integrated downstream firm is followed by the infinitely repeated play of the “noncollusive equilibrium” (which is a subgame-perfect equilibrium of the stage game). In contrast,

$^6$ We assume that holding fixed downstream rivals’ prices, $D_j$’s demand is positive if it charges a sufficiently low price, and weakly decreasing in its own price (and strictly decreasing if $D_j$’s demand is positive). Holding fixed its own price, $D_j$’s demand is weakly increasing in downstream rival $D_i$’s price (and strictly increasing if both $D_i$ and $D_j$ face positive demand). We assume that demand is such that in the associated one-shot simultaneous-move game in which $N$ (downstream) firms compete in prices (respectively, quantities) with downstream firm $D_j$ facing a constant marginal cost (wholesale price) $w_i$, there is a unique Nash equilibrium outcome. See Xavier Vives (1999) for conditions on demand that ensure our assumptions.

$^7$ Downstream firms are allowed to accept more than one offer, i.e., contracts are nonexclusive.

$^8$ Suppose downstream firm $D_j$ were to set a price $p_j$ at which $Q(p_j; p_{-j}) > 0$ (or, under quantity competition, that $D_j$ sets $q_j > 0$), but $D_j$ later rejects all of its offers, and so is unable to satisfy consumer demand. For completeness, we assume that, in this case, the rationed consumers adjust their demand for the rival final goods accordingly, and so demand for $D_j$’s rivals is as if $p_j = \infty$ (or $q_j = 0$).
deviations by unintegrated downstream firms do not trigger any punishment.\footnote{Since we are concerned with upstream collusion, we focus on those equilibria of the repeated game that are best from the upstream firms’ point of view. Deviations by unintegrated downstream firms will not be punished, as these firms would actually benefit from triggering such punishments. But an integrated firm’s upstream and downstream affiliates share the same objective function, and so the punishment phase is triggered not only when an upstream firm deviates but also when an integrated downstream firm deviates. In contrast, Hans Theo Normann (2006)—analyzing whether vertical foreclosure may be sustainable in a repeated game—considers the case where an integrated firm will be punished either for upstream or downstream deviations but not both.}

We define the \textit{critical discount factor} \(\delta\) as the lowest value of \(\delta\) such that, for all \(\delta \geq \delta\), there exists an equilibrium in which all of the monopoly rents are extracted by upstream firms. As is common in the industrial organization literature, we will say that a vertical merger \textit{facilitates} upstream collusion if it \textit{reduces} the critical discount factor \(\delta\).

Given any market structure, minimizing the critical discount factor entails all the upstream firms having the same net incentive to deviate. Integer constraints will, however, generally complicate this equalization of deviation incentives in a pure-strategy collusive equilibrium if downstream firms produce differentiated goods. (Upstream offers will generally involve positive fixed fees, and so each unintegrated downstream firm will typically accept at most one offer.) The existence of a public correlating device \(\theta\) allows us to abstract from this issue. Along the collusive equilibrium path, the realization of \(\theta\) determines downstream firms’ choice among the upstream firms’ offers over which they are indifferent. Since all upstream and downstream firms make their pricing decisions before \(\theta\) is realized, the realization of \(\theta\) does not affect firms’ incentives to deviate.\footnote{Instead of using a public correlating device, upstream firms may share collusive profits by making side payments at the end of each period. Failure to make side payments will trigger the punishment phase. Side payments need not be made following a deviation at the pricing stage. It can be shown that if no firm has an incentive to deviate at the pricing stage, then no firm has an incentive to deviate by not making the required side payments.}

\section{Equilibrium Analysis}

\subsection{Noncollusive Equilibrium}

The stage game has multiple subgame-perfect equilibria. Here, we focus on the \textit{symmetric noncollusive equilibrium}.\footnote{See Nocke and White (2005) for a characterization of all subgame-perfect equilibria of the stage game and a justification for our focus on the symmetric equilibrium.} We assume that firms will revert to the infinite play of this equilibrium after a deviation. In this equilibrium, each upstream firm makes offers of the form \((w_i, F_i) = (0, 0)\) to each downstream firm, and each downstream firm chooses the corresponding noncollusive price \(p^{NC}\) under price competition (or quantity \(q^{NC}\) under quantity competition), where \(p^{NC} = \arg \max_i pQ(p; p^{NC}, \ldots, p^{NC})\) and \(q^{NC} = \arg \max_i qP(q; q^{NC}, \ldots, q^{NC})\). At the acceptance stage, each unintegrated downstream firm accepts at least one contract along the equilibrium path. In this equilibrium, each (unintegrated) upstream firm makes zero profit, and each downstream firm makes a profit of \(\pi^{NC} \geq 0\), where the inequality is strict unless final goods are homogeneous and downstream competition is in prices. Since final goods are substitutes, industry profits in the symmetric noncollusive equilibrium are less than monopoly profits, \(N\pi^{NC} < II\).

The symmetric noncollusive equilibrium is the natural generalization of the standard Bertrand equilibrium to two-part tariffs. Since each (unintegrated) upstream firm makes zero profit in this equilibrium, and any integrated firm faces downstream rivals that all obtain the input at zero cost, this equilibrium minimizes the profit of each unintegrated upstream firm and each integrated firm in the set of subgame-perfect equilibria of the stage game. Finally, note that the allocation in the symmetric noncollusive equilibrium is independent of the number of integrated upstream-downstream pairs. This feature of our setup is useful since it allows us to focus attention on the \textit{collusive} effects of vertical integration.

\subsection{Collusive Equilibrium: Nonintegration}

We now consider the collusive equilibrium when no firm is vertically integrated. We focus on the collusive equilibrium where the upstream firms jointly extract all of the monopoly rents...
\( \Pi^M \) for the monopoly rents to be generated, each downstream firm \( D_j \) must, at the pricing stage, set the monopoly price \( p_j = p^M \) (under price competition) or the monopoly quantity \( q_j = Q^M/N = q^M \) (under quantity competition).\(^{12}\) At the same time, each of the \( M \) upstream firms makes the same offer \((w^M, F^M)\) to each of the \( N \) downstream firms. The collusive wholesale price \( w^M \leq p^M \) is chosen such that it is a best response for each downstream firm \( D_j \), to charge the price \( p^M \) (or produce the quantity \( q^M \)), given the equilibrium behavior of its \( N - 1 \) downstream rivals. The fixed fee \( F^M \equiv 0 \) is chosen so as to extract all of the rents from a downstream firm. If final goods are homogeneous and downstream competition is in prices, \((w^M, F^M) = (p^M, 0)\); otherwise, there is double marginalization, and so \( w^M < p^M \) and \( F^M > 0 \).\(^{13}\) At the acceptance stage, each downstream firm accepts one offer along the equilibrium path.\(^{14}\) At the output stage, each downstream firm will then face a demand of \( q^M = Q(p^M; p^M, \ldots, p^M) \), which it will order from its upstream supplier at wholesale price \( w^M \). Since all upstream firms are symmetric under nonintegration, the critical discount factor is minimized if the upstream firms share the market equally. Hence, along the equilibrium path, each upstream firm receives an expected per-period profit of \( \Pi^M/M \).

Consider now an upstream firm’s incentive to deviate.\(^{15}\) By offering the contract \((w^M - \epsilon, F^M - \epsilon)\) to each of the \( N \) downstream retailers, where \( \epsilon \) is arbitrarily small, the deviator can obtain a deviation profit arbitrarily close to \( \Pi^M \). Each downstream firm would find it profitable to accept such an offer, independently of the acceptance decisions of its downstream rivals, and to order all units demanded by consumers from the deviator. Production and pricing will be at monopoly levels. This (or any other) out-of-equilibrium contract offer by an upstream firm triggers a switch to the punishment phase in which firms play the noncollusive equilibrium in all future periods and, as discussed above, upstream firms receive zero profit. Notice that upstream profits from deviation and punishment do not depend on whether downstream competition is in prices or quantities, or indeed whether goods are differentiated or homogeneous (except insofar as the latter affects the level of monopoly profit). So, independently of the form that downstream competition takes, under nonintegration, an upstream firm’s “no-cheating” incentive constraint can be written as

\[
\frac{\Pi^M}{M(1 - \delta)} \geq \Pi^M.
\]

The left-hand side of this expression represents the discounted sum of profits along the collusive equilibrium path, and the right-hand side represents the maximum profit a deviant upstream firm can obtain. Hence, the critical discount factor under nonintegration (NI) is given by

\[
\delta^{NI} = \frac{M - 1}{M}.
\]

Intuitively, a deviant upstream firm can always sever the ties between its upstream rivals and the unintegrated downstream firms by slightly undercutting rival offers, allowing it to obtain (arbitrarily close to) the collusive industry profit in the period of deviation. In fact, this holds not only for the monopoly profit \( \Pi^M \), but more generally for any upstream industry profit level \( \Pi \leq \Pi^M \). Hence, under nonintegration, any positive upstream industry profit level \( \Pi \in (0, \Pi^M] \) can be sustained as an equilibrium if and only if \( \delta \geq \delta^{NI} \).

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\(^{12}\) If final goods are homogeneous, any vector of downstream outputs that adds up to \( Q^M \) can be induced by appropriate contracts and be used to implement the monopoly outcome. The incentives to deviate set out below are unaffected by the way in which the production of \( Q^M \) is “shared” between downstream firms.

\(^{13}\) If downstream competition is in prices, we have \( w^M = p^M + [Q(p^M; p^M, \ldots, p^M)/Q(p^M; p^M, \ldots, p^M)](F^M - w^M) = -[Q(p^M; p^M, \ldots, p^M)/Q(p^M; p^M, \ldots, p^M)](F^M - w^M) \) and \( F^M = [p^M - w^M](Q(p^M; p^M, \ldots, p^M)/Q(p^M; p^M, \ldots, p^M)) \), which denotes the derivative of \( Q \) with respect to its first argument. A similar expression obtains under quantity competition.

\(^{14}\) When \( N \) is not an integer multiple of \( M \), downstream firms use the public randomization device to determine their acceptances.

\(^{15}\) To ensure that downstream firms have no incentive to deviate, we assume that if downstream firm \( D_j \) deviates by setting a price \( p_j \) (or a quantity \( q_j \)), then the
C. Collusive Equilibrium: Single Integration

We now turn to the collusive equilibrium when one upstream-downstream pair, say $U_i-D_1$, is vertically integrated. Since the market structure is no longer symmetric, it may be optimal for upstream collusion to allow the integrated $U_i-D_1$ to capture a share of the collusive equilibrium profit that is different from (in fact, larger than) that of the $M-1$ unintegrated upstream rivals. Let $\alpha$ denote the collusive market share of the integrated $U_i-D_1$. Symmetry of the $M-1$ unintegrated upstream firms implies that the critical discount factor is minimized if each of them obtains the same share, $(1-\alpha)/(M-1)$, of the collusive equilibrium profit. At the pricing stage, the collusive equilibrium behavior is as outlined above for the case of nonintegration: each upstream firm offers the contract $(w^M,F^M)$ to each of the (unintegrated) downstream firms, and each downstream firm charges a price of $p^M$ (or sets a quantity of $q^M$). At the acceptance stage, the acceptance/rejection decisions of the downstream firms will, in expectation, reflect the market-sharing arrangement under vertical integration.

Consider first the integrated $U_i$’s incentive to deviate. Along the equilibrium path, the firm obtains an expected per-period profit of $\alpha \Pi^M$. By offering the deviant contract $(w^M-e,F^M-e)$ to each of the $N-1$ unintegrated downstream firms, where $e$ is arbitrarily small, the integrated firm can obtain their business and make a profit arbitrarily close to the monopoly profit $\Pi^M$ in the period of deviation.\footnote{The integrated $U_i-D_1$ cannot extract more than the monopoly profit $\Pi^M$ in the period of deviation. To do so, some other firm would need to make a loss in the period of deviation. But each downstream firm can ensure itself a profit of zero by rejecting all contracts. Finally, since all upstream firms make equilibrium offers involving nonnegative fixed fees and wholesale prices, each upstream firm’s profit must be larger than or equal to zero.} Following the integrated firm’s deviation, firms coordinate on the (symmetric) noncollusive equilibrium in all future periods. In this noncollusive equilibrium, the integrated firm makes a per-period profit of $\pi^{NC}$ through its downstream affiliate $D_1$. The integrated firm’s incentive constraint is thus given by:

$$\frac{\alpha \Pi^M}{1-\delta} \geq \Pi^M + \frac{\delta}{1-\delta} \pi^{NC}.$$  

Comparing this equation with an upstream firm’s incentive constraint (2) under nonintegration, we see that there is an additional term on the right-hand side of (4), $\delta \pi^{NC}/(1-\delta)$. This term represents the punishment effect of vertical integration: it is more difficult to punish an integrated firm than an unintegrated upstream firm. Unless downstream products are homogeneous and retail competition is in prices, the integrated downstream affiliate makes positive profits in the punishment phase, $\pi^{NC} > 0$.

Consider now an unintegrated $U_i$’s incentive to deviate, $i \geq 2$. There is no punishment effect for $U_i$ since, in periods following a deviation, all unintegrated upstream firms make zero profits, as in the absence of integration. Along the equilibrium path, $U_i$ obtains a per-period profit of $(1-\alpha)\Pi^M/(M-1)$. By offering instead the deviant contract $(w^M-e,F^M-e)$ to each of the $N-1$ unintegrated downstream firms, for arbitrarily small $e$, $U_i$ can gain the business of all the unintegrated downstream firms, and extract (arbitrarily close to) $\Pi^M/N$ from each one of them, as above. Importantly, however, the deviant $U_i$ will not be able to extract any profit from the integrated $D_1$. Since $D_1$ can obtain the intermediate input at zero marginal cost from its own upstream affiliate, $U_i$, $D_1$ will not accept any (deviant) contract that does not leave $D_1$ all of the rents: it will always be at least as cheap for $D_1$ to obtain the input from $U_i$. Vertical integration therefore reduces the deviation profit of an unintegrated upstream firm by $\Pi^M/N$: the amount that it would have made from selling to $D_1$ if $D_1$ were not integrated. This is what we call the outlets effect of vertical integration. Comparing with equation (2) above, we can clearly see how the outlets effect slackens the incentive constraint of an unintegrated upstream firm:

$$\frac{(1-\alpha)\Pi^M}{(M-1)(1-\delta)} \geq \Pi^M - \frac{1}{N} \Pi^M.$$  

We now assume that, if the discount factor is low enough to make it necessary for collusion, upstream firms set market shares in such a way as
to minimize the collective incentive to deviate.\textsuperscript{17} This assumption allows us to add up the incentive constraint (4) for the integrated firm and the $M - 1$ incentive constraints (5) for the unintegrated upstream firms to obtain the minimum collective incentive to deviate. Rearranging and replacing the critical discount factor under single integration (SI):\textsuperscript{18}

$$\delta^{SI} = \frac{M - 1}{M + \frac{\Pi^M - N\pi^{NC}}{(N - 1)\Pi^M}}. $$

D. The Collusive Effect of Vertical Integration

Comparing equations (3) and (6), we obtain our main result:

**PROPOSITION 1:** In a vertically unintegrated industry, a vertical merger facilitates upstream collusion: $\delta^{SI} < \delta^{NI}$.\textsuperscript{18}

A vertical merger between an upstream and a downstream firm has two opposing effects on upstream firms’ incentives to collude. On the one hand, an unintegrated upstream firm cannot profitably deviate through a rival’s integrated downstream affiliate; this outlets effect reduces the unintegrated firm’s deviation profit (by $\Pi^M/N$), and hence its incentive to deviate. On the other hand, an integrated firm captures the profit of its downstream affiliate (which is $\pi^{NC} < \Pi^M/N$) in the punishment phase, while all unintegrated upstream firms make zero profit in the punishment phase, independently of market structure. This punishment effect increases the integrated firm’s incentive to deviate, holding fixed its market share.

Which effect is stronger? Following a single vertical merger, there are $M - 1$ remaining unintegrated upstream firms, and so the combined outlets effect reduces overall deviation profits by $(M - 1)\Pi^M/N$. The punishment effect increases the integrated firm’s discounted sum of profits from deviating by $\delta\pi^{NC}/(1 - \delta)$, holding fixed the integrated firm’s market share. Under non-integration, the critical discount factor is $\delta^{NI} = (M - 1)/M$. Evaluated at $\delta = \delta^{NI}$, however, we have $\delta\pi^{NC}/(1 - \delta) = (M - 1)\pi^{NC} < (M - 1)\Pi^M/N$. Hence, the outlets effect outweighs the punishment effect since $\pi^{NC} < \Pi^M/N$. This also shows that if final goods were completely independent, and so $\pi^{NC} = \Pi^M/N$, a vertical merger would have no collusive effect.

E. Other Collusive Equilibria

We have shown that a vertical merger facilitates upstream collusion in the sense that it reduces the critical discount factor above which monopoly profits upstream can be sustained. One might wonder about the effect of vertical merger on upstream firms’ ability to sustain profits above noncooperative levels, but still less than those associated with monopoly prices. In this subsection, we argue that vertical merger also helps sustain these other collusive outcomes, and that this should be of concern for antitrust authorities.

There are two reasons why firms may wish to sustain outcomes that are “less collusive” than monopoly. First, it may be that the monopoly outcome itself is not feasible and, second, it may be that while the monopoly outcome is feasible, the division of the pie required to sustain the monopoly outcome is not favorable for some particular firm(s). We now deal with each of these issues in turn. In each case, since we are concerned here with upstream collusion, we will focus on equilibria that lie on upstream firms’ Pareto frontier.

First, would a vertical merger harm firms’ ability to sustain lower levels of collusive profit when monopoly itself is not feasible? The answer to this turns out to be negative. As we have shown in Section IIIIB, when no firm is vertically integrated, positive upstream profits are sustainable if and only if monopoly profits for
upstream firms are sustainable. Proposition 1 then implies that, holding the discount factor fixed, the aggregate level of upstream profit that can be sustained under single integration is no lower than that which can be sustained without vertical integration, and that, further, there is a range of discount factors for which positive profits can be sustained if and only if one upstream-downstream pair is vertically integrated. This range is larger than it would appear from comparing $\delta^{NI}$ and $\delta^{SI}$ since, when one upstream-downstream pair is vertically integrated, the feasibility of collusion does not have the same all-or-nothing feature that it has in the absence of vertical integration. It turns out that vertical integration also facilitates sustaining imperfect collusion, making it possible to sustain positive profits below monopoly level even when the discount factor is too low for monopoly outcomes to be sustainable. (We give an example of such an equilibrium in footnote 19 below.)

Now let us turn to the second concern—the idea that, even when monopoly outcomes are sustainable, firms might nevertheless prefer to implement some other collusive scheme. Clearly, for the reasons noted above, this is not a concern in the unintegrated case. For the case when one firm is integrated, one can also show that the unintegrated upstream firms are better off in their preferred equilibrium involving monopoly profits upstream than they are in any other collusive equilibrium. The integrated $U_1-D_1$, however, may be better off in an equilibrium in which upstream industry profits are less than $\Pi^{M}$. To see this, consider the thought experiment of increasing each unintegrated $D_j$’s price from $p^M$ to $p^j > p^M$ (or reducing $D_j$’s quantity from $q^M$ to $q^j < q^M$), which amounts to increasing the market share of the integrated $D_1$. While this reduces the industry profit, it strengthens the outlets effect because the fraction of the industry profit that a deviant unintegrated upstream firm can capture is now reduced to less than $(N - 1)/N$. Hence, the integrated $U_1-D_1$ may be better off since it may be able to obtain a larger share of the (albeit smaller) collusive pie.\(^\text{19}\)

\(^{19}\)This point can best be seen by considering the extreme case when final goods are almost perfect substitutes. If each unintegrated $D_j$ sets $p_j = \infty$ (or $q_j = 0$), while the

Our main result (Proposition 1) suggests that antitrust regulators should be wary of the collusive effects of vertical merger. We argue that this remains true if, following a vertical merger, firms elect to play a collusive equilibrium with a symmetric downstream output preferred by the integrated firm, because welfare (as measured by total surplus) will tend to be lower in such an asymmetric collusive equilibrium than under monopoly. Given that final goods are symmetrically differentiated, any asymmetry in retail pricing introduces an additional efficiency loss. So when both are feasible, collusion under a vertically integrated structure can easily be strictly worse than collusion under an unintegrated structure if the former involves asymmetric downstream production. We verify that this is indeed the case for our linear demand example.

**EXAMPLE 2 (Linear Demand):** Suppose there are two upstream and two downstream firms, $M = N = 2$, downstream competition is in prices, and one upstream-downstream pair, say $U_1-D_1$, is vertically integrated. It can be shown that if monopoly profits upstream are sustainable, then in any equilibrium on upstream firms’ Pareto frontier, total surplus is less than or equal to total surplus under monopoly. For instance, consider the equilibrium that maximizes the integrated $U_1-D_1$’s profit. If final goods are sufficiently good substitutes, $\sigma \geq \tilde{\sigma} = 2\delta(1 + \delta)$, then $p_1 = p^M$ and $p_2 = \infty$; obviously, welfare is lower than under monopoly. Otherwise, if $\sigma < \tilde{\sigma}$, then $p_1 < p^M < p_2$, and demand is such that $q_1 + q_2 < 2q^M = Q^M$; again, total surplus is lower than under monopoly.

Thus, vertical merger facilitates not only the maintenance of monopoly prices, but also the maintenance of lower levels of collusive profits. Moreover, when collusion is sustainable even in
the absence of integration, vertical integration may result in the adoption of an asymmetric market structure which has lower total surplus than the monopoly outcome.

F. Incentives for Vertical Mergers

According to the Chicago School of antitrust, firms have no (strict) incentive to integrate vertically if there are no efficiency gains from doing so. In our model, a vertical merger has no direct efficiency effects. Accordingly, in the symmetric noncollusive equilibrium of our game, firms have no incentive to merge vertically: the joint profit of any upstream-downstream pair in the symmetric noncollusive equilibrium is $\pi_{NC}$, independently of whether or not the pair is vertically integrated. Nevertheless, in our model, an upstream-downstream pair has two distinct but related motives to merge vertically: a collusive motive and a market share motive.

The collusive motive for vertical merger arises since upstream collusion may be sustainable only if an upstream-downstream pair vertically integrates. As we have shown above, if $\delta^{NI} \leq \delta < \delta^{NI}$, then by integrating, an upstream-downstream pair can make upstream collusion sustainable where it was not under nonintegration. Integration raises the joint per-period profit of the integrating firms from $\pi_{NC}$ to $\alpha \Pi^M \geq (1 - \delta) \Pi^M + \delta \pi_{NC} > \pi_{NC}$. At the same time, it also raises the joint profits of the unintegrated upstream firms from zero to $(1 - \alpha) \Pi^M$, even though their combined market share falls.20

But a vertical merger can also hurt unintegrated upstream rivals, in particular when upstream collusion is feasible even in the absence of vertical integration, $\delta \geq \delta^{NI}$. In this case, there is a market share motive for vertical merger. Under nonintegration, there is at least one upstream-downstream pair with a per-period profit not larger than $\Pi^M$. By vertically integrating, however, this upstream-downstream pair can ensure itself a collusive per-period profit of at least $\alpha \Pi^M \geq (1 - \delta) \Pi^M + \delta \pi_{NC}$, since otherwise the integrated firm would have an incentive to cheat. Other firms must concede this increase in market share for collusion not to break down. Vertical integration thus increases the newly integrated upstream-downstream pair’s profit if $(1 - \delta) \Pi^M + \delta \pi_{NC} > \Pi^M / M$, or $\delta < (M - 1) \{ M[1 - \pi_{NC} / \Pi^M] \} \equiv \delta$. It can easily be verified that $\delta > \delta^{NI}$ if $\pi_{NC} > 0$. Hence, if $\delta^{NI} \leq \delta < \delta$, a vertically integrated upstream-downstream pair must obtain a larger share of the collusive profit for upstream collusion to be sustainable, and this pair has a market share motive for vertical merger. In fact, many case studies of cartels report instances of vertically integrated firms demanding and receiving larger shares of the collusive pie (e.g., Levenstein 1997; Toldsal 1917).

G. Multiple Integration

We have shown above that vertical integration by a single upstream-downstream pair facilitates upstream collusion relative to the case where no firm is vertically integrated. Does further integration also facilitate collusion? To address this question, we take the same model as before, continuing to focus on collusive equilibria that sustain the full monopoly outcome, but now suppose that $K \leq \min \{ M, N \}$ upstream-downstream pairs (say, $U_1-D_1$ to $U_K-D_K$) are vertically integrated.21

Contract offers and downstream prices/quantities along the collusive equilibrium path are as outlined above for the case of single vertical integration. Let $\alpha$ denote the share of the monopoly profit that the $K$ vertically integrated firms jointly obtain along the collusive equilibrium path. The no-cheating constraints for an unintegrated upstream firm and an integrated upstream-downstream pair can then be written as

\[
\frac{(1 - \alpha) \Pi^M}{(M - K)(1 - \delta)} \geq \left( \frac{N - K}{N} \right) \Pi^M
\]

20 From the incentive constraints, the market share of the integrated firm must then satisfy $\alpha > 1 / M$. This is because vertical integration makes collusion feasible by slackening the unintegrated firms’ incentive constraint (through the outlets effect), while tightening the integrated firm’s incentive constraint (through the punishment effect), so the integrated firm’s share must rise relatively.

21 Unlike the literature on sequential mergers, we do not consider a sequential timing of mergers but rather analyze the comparative statics of collusive equilibria with respect to $K$. It would also be interesting to analyze the case where an upstream firm vertically integrates with more than one downstream firm. We assume, however, that this would be prohibited by antitrust authorities as it would entail an element of horizontal merger.
and

\[
\frac{\alpha \Pi^M}{K(1 - \delta)} \geq \pi^\text{dev}_{\text{int}}(K) + \frac{\delta}{1 - \delta} \pi^\text{NC},
\]

respectively. In (8), \(\pi^\text{dev}_{\text{int}}(K)\) denotes the maximum deviation profit of an integrated upstream-downstream pair. Summing up the incentive constraints for the unintegrated and integrated upstream firms, (7) and (8), we obtain the critical discount factor with \(K\) vertically integrated firms:

\[
\hat{\delta}(K) = \frac{(M - K)(N - K)\Pi^M + N[K\pi^\text{dev}_{\text{int}}(K) - \Pi^M]}{(M - K)(N - K)\Pi^M + NK[\pi^\text{dev}_{\text{int}}(K) - \pi^\text{NC}]}.
\]

Does the outlets effect outweigh the punishment effect for each vertical integration, so that \(\hat{\delta}(K)\) is decreasing in \(K\)? The answer is, not necessarily. First, note that while the punishment effect is of the same size as before, the outlets effect is smaller for an integrated firm than for an unintegrated upstream firm, \(\pi^\text{dev}_{\text{int}}(K - 1) > \pi^\text{dev}_{\text{int}}(K) < \Pi^M/N\) for \(K > 1\), since an integrated firm can increase its deviation profit by changing its own downstream price to steal business from integrated downstream rivals that way. Second, while the vertical merger between \(U_k\) and \(D_k\), \(K > 1\) reduces the incentives to cheat for the other \(K - 1\) already integrated firms, \(U_1 - D_1\) to \(U_{K-1} - D_{K-1}\), through the outlets effect, it increases \(U_k\)'s profit in the period of deviation. Effectively this is because, by integrating, \(U_k\) can coordinate its upstream deviation with a downstream price reduction, which it could not before. For a single vertical merger, this coordination was not necessary and the effect did not occur: the deviation profit of an unintegrated upstream firm under nonintegration is the same as that of the integrated \(U_1 - D_1\) under single integration, namely \(\Pi^M\). But with \(K - 1\) vertically integrated firms, the deviation profit of the unintegrated \(U_k\) is \((N - K + 1)\Pi^M/N\), which is less than \(U_k\)'s deviation profit after integration, \(\pi^\text{dev}_{\text{int}}(K)\).\(^{22}\)

It is instructive to consider some limiting cases. First, suppose final goods become almost perfect substitutes and downstream competition is in prices, and so \(\pi^\text{NC} = 0\) and \(\pi^\text{dev}_{\text{int}}(K) \approx \Pi^M\). In this case, the critical discount factor becomes

\[
\hat{\delta}(K) \approx \frac{M(N - K) + K^2 - N}{M(N - K) + K^2},
\]

which is minimized at \(K^* = M/2\), i.e., when half of the upstream firms are vertically integrated. Our theory thus helps us to understand why a limited degree of vertical merger may be profitable in industries aiming to collude. This is interesting since many industries seem to have the feature that vertically integrated firms compete with separated ones, and it is not always clear why such differing arrangements should arise.\(^{23}\)

Next, consider the thought experiment of increasing the number \(N\) of downstream firms, holding fixed \(M\). In the limit, industry profits in the noncollusive equilibrium vanish (relative to the monopoly profit), \(N\pi^\text{NC}/\Pi^M \to 0\). Moreover, since a deviant integrated firm can extract the rent of the \(N - K\) unintegrated firms, it will only slightly reduce its own downstream price (or just slightly increase its own downstream quantity) when deviating so that it does not reduce the unintegrated downstream firms’ rent too much, and so \(\pi^\text{dev}_{\text{int}}(K) \to [(N - K + 1)/N]\Pi^M\). As the following proposition shows, for \(N\) large, the critical discount factor is minimized when all \(M\) upstream firms are vertically integrated. Consider now the thought experiment of increasing the number \(M\) of upstream firms, holding fixed \(N\). For \(M\) large, the critical discount factor is minimized when all \(N\) downstream firms are vertically integrated, as this effectively permits the integrated firms to exclude the remaining \(M - N\) unintegrated upstream firms from obtaining any of the collusive pie. Finally, the proposition also shows that, for any \(K \geq 1\), the \(K\text{th}\) vertical integration facilitates collusion as long as the number of upstream and downstream firms is sufficiently large.

This would result in the same deviation profit as \(U_k\)'s deviation profit prior to vertical integration, \((N - K + 1)\Pi^M/N\). However, if \(K > 1\), and unless final goods are homogeneous and downstream competition is in prices, it will always be optimal for the integrated \(U_k - D_k\) to set \(p_k\) below the monopoly price \(p^M\), and so \(\pi^\text{dev}_{\text{int}}(K) > (N - K + 1)\Pi^M/N\).\(^{23}\) For examples, see Kirsten Bindemann (1999) on the oil industry, Christopher Woodruff (2002) on the Mexican footwear industry, Margaret E. Slade (1998a, b) on the UK beer industry and the gasoline retail market in Vancouver, respectively, and Tasneem Chipty (2001) and David Waterman and Andrew A. Weiss (1996) on the US cable television industry. For an alternative theoretical rationale for asymmetric outcomes, see Ordover, Saloner, and Salop (1990).

\(^{22}\)To see this, note that the integrated \(U_k - D_k\) can always deviate by not changing its own downstream affiliate's price.

\(^{23}\)Andrew A. Weiss (1996) on the US cable television industry.
Suppose $N\Pi^N/\Pi^M \to 0$ and $\lim_{N \to \infty} \frac{1 - \pi_{\text{int}}(K)}{\Pi^M} = K - 1$ as $N \to \infty$. (i) Holding $M$ fixed, each merger of an upstream-downstream pair facilitates collusion for $N$ sufficiently large. (ii) Holding $N$ fixed, each merger of an upstream-downstream pair facilitates collusion for $M$ sufficiently large. (iii) Consider any sequence $\{M_i, N_i\}_{i=1}^\infty$ where $M_{s+1} \geq M_s$, $\lim_{s \to \infty} M_s = \infty$, $N_{s+1} \geq N_s$, $\lim_{s \to \infty} N_s = \infty$, and $\lim_{s \to \infty} M_s/N_s = \mu \in (0, \infty)$. For any $K \geq 1$, the $K^{th}$ vertical integration facilitates collusion for $s$ sufficiently large.

The relationship between the degree of product differentiation and the number of vertical mergers that minimizes the critical discount factor is illustrated in the following example.

**EXAMPLE 3 (Linear Demand):** Suppose downstream competition is in prices and there are three upstream firms, $M = 3$. When $N = 3$, the critical discount factor is minimized with three vertical mergers, $K^* = 3$, if goods are sufficiently differentiated, $\sigma \in (0, 0.53452)$; and with two vertical mergers, $K^* = 2$, if goods are sufficiently close substitutes, $\sigma \in (0.53452, 1)$. The threshold degree of substitutability between $K^* = 3$ and $K^* = 2$ increases as the number of downstream firms increases: it is $\sigma = 0.66117$ for $N = 4$, $\sigma = 0.87413$ for $N = 10$, and converges to 1 as $N \to \infty$.

The result that the critical discount factor may be minimized with an intermediate number of vertical mergers indicates that the reason why vertical mergers facilitate upstream collusion is not that collusion may be easier to sustain downstream than upstream.\(^{24}\) Instead, the result is due to the relative sizes of the outlets and punishment effects, as discussed above. The example also shows that, while the interplay of these effects is more complicated for the multiple integration case, given a demand function and a market structure, our model allows the collusive impact of any particular vertical merger to be calculated. Hence it provides an important tool for the antitrust analysis of vertical mergers.

\(^{24}\) In fact, if $N$ is much larger than $M$, then, in the absence of vertical integration, it is easier to sustain collusion upstream than downstream.

**H. Robustness**

**Secret Offers.**—In our analysis, we have assumed that all prices, including upstream firms’ contract offers, are publicly observable. Would our results still hold in the plausible case where downstream firms, at the acceptance stage, cannot observe the details of their rivals’ contract offers? Intuitively, the answer is “yes” as, in our model, downstream prices are set publicly before acceptance decisions on contract offers are made, and downstream firms care about their rivals’ prices or quantities, not, per se, about the wholesale prices and fixed fees their rivals face. In the Web Appendix, we show that our main result—that a (single) vertical merger facilitates upstream collusion—carries over to the case of secret offers. In our analysis, we impose a plausible restriction on downstream firms’ out-of-equilibrium beliefs—so-called “wary beliefs” (R. Preston McAfee and Marius Schwartz 1994). These beliefs are required only in the subgame in which the integrated $U_1$ has deviated by changing the price (or quantity) of its downstream affiliate $D_1$.

**Optimal Punishment.**—In our analysis, we have assumed that the deviation of an upstream firm triggers a reversion to the noncollusive equilibrium in all subsequent periods. Although unintegrated upstream firms make zero profit in the noncollusive equilibrium, this punishment scheme is not optimal: to sustain collusion for some discount factors, it may be necessary to provide incentives for downstream firms to reject deviant offers when they are made. In the Web Appendix, we derive the optimal punishment scheme for the case when final goods are homogeneous, and show that a (single) vertical merger still facilitates upstream collusion. Our analysis is of independent technical interest as it reveals that the logic of simple penal codes (Dilip Abreu 1988) breaks down in repeated extensive-form games: for some discount factors, collusion may be sustainable only by a strategy profile with the property that the “punishment” not only depends on the identity of the deviator but is also “fined-tuned” to the details of the deviation made.\(^{25}\)

\(^{25}\) This last point is further discussed in George J. Mailath, Nocke, and White (2004).
Entry.—Our focus on equilibria where upstream firms extract the entire monopoly rents from collusion would be restrictive in an augmented model where excessive profits upstream attract entry, and also in a world where buyers need to be rewarded for making investments, or could react to upstream collusion by trying to establish an independent source of supply. Would our results be robust to these empirically reasonable extensions? While these topics are outside the scope of our paper, we believe that the same basic arguments would go through: the outlets and punishment effects would still arise in this richer framework. The implication of each of these extensions would be that the downstream firms must share in the rents from collusion. But from an industry point of view, it is still desirable to coordinate on contracts that implement the monopoly outcome; it is only that the fixed fee charged by upstream to downstream firms should be lower in order to redistribute some rents. One would need to be careful, though, to consider how, if downstream firms earn rents, they can best be involved in the collusive scheme. Our analysis in the Web Appendix of the optimal punishment scheme gives an idea of how complicated such involvement can become: downstream firms are offered rents for rejecting deviant offers that it would otherwise be profitable for them to accept. If what one ultimately cares about is how vertical integration might facilitate coordination between firms at each level of the vertical hierarchy, one can view our analysis as a first step along this road. But we leave this as a direction for future research.\textsuperscript{26}

IV. A Model with Sequential Moves

Thus far, we have assumed that upstream contract offers and downstream retail prices are chosen simultaneously. In many circumstances, however, it seems plausible that downstream firms have contracts with their input suppliers in place before deciding upon their own output prices. In this section, we analyze the robustness of our predictions to a change in the sequence of moves. In particular, we assume the following timing in each period:

- **Contract offer stage:** Upstream firms $U_1, \ldots, U_M$ simultaneously make public two-part tariff contract offers to the downstream firms.
- **Sunspot stage:** A public random variable $\theta$, uniformly distributed on $[0,1]$, is realized.
- **Acceptance stage:** Downstream firms $D_1, \ldots, D_N$ simultaneously decide which contract(s) to accept.
- **Downstream pricing stage:** Downstream firms $D_1, \ldots, D_N$ simultaneously set prices (or quantities) in the retail market, and then order the quantities demanded by consumers from the upstream firms at the relevant wholesale prices.\textsuperscript{27}

As before, we will analyze the impact of a single vertical merger on the critical discount factor, confining attention to a punishment scheme that involves infinite reversion to the (symmetric) noncollusive equilibrium. It turns out that the change in timing does not upset this noncollusive equilibrium outcome, nor the contracts implementing the monopoly outcome with or without vertical integration.\textsuperscript{28} Thus, one can show that the critical discount factor under non-integration is as shown in Section IIIB above,

$$\delta_{\text{seq}}^N = (M - 1)/M.$$  

Therefore, we can turn directly to the collusive equilibrium when one upstream-downstream pair, say $U_1-D_1$, is vertically integrated. As we

\textsuperscript{26}We are not aware of any papers studying collusion among an entire vertical hierarchy. In fact, the study of collusive equilibria where buyers are interdependent and behave strategically is in its infancy. Apart from this paper, the only other paper we know of is Christopher M. Snyder (1996), who examines collusion in the presence of a single strategic buyer.

\textsuperscript{27}If downstream firm $D_j$ has rejected all contract offers at the acceptance stage, it is forced to set $p_j = \infty$ (or quantity $q_j = 0$) at the downstream pricing stage, and thus make zero profit.

\textsuperscript{28}With the sequential timing, we need to be more careful about how the collusive profits are shared among upstream firms when they use the public randomization device. Under vertical integration, it may be necessary for upstream collusion that the integrated $U_1-D_1$’s profit be independent of the outcome of the randomization device so as not to interfere with $U_1-D_1$’s incentive constraint at the pricing stage. This can be achieved by prescribing that only a fraction of the integrated $D_j$’s equilibrium output is ordered from its upstream affiliate $U_j$, the remaining units being ordered from other upstream firms. It can easily be verified that such a cross-selling arrangement does not affect upstream firms’ incentives to offer deviant contracts. Alternatively, as before, the issue can be resolved by using side payments at the end of the period.
will see, the outlets and punishment effects are still present with this modified timing, but the flexibility of downstream pricing (quantity setting) results in two further effects on deviation profits, which we call the reaction effect and the lack-of-commitment effect. As we now show, both these effects reduce deviation profits and hence make upstream collusion easier to sustain.

Consider first the incentive to deviate of an unintegrated upstream firm, \( U_i \), \( i \geq 2 \). As in the model with simultaneous moves, the unintegrated \( U_i \) can extract no rents from offering a deviant contract to the integrated downstream firm \( D_1 \). So, if the integrated \( U_i-D_1 \)’s retail price were fixed at the collusive price \( p^M \) (or its quantity were fixed at \( q^M \)), then the deviant \( U_i \) could obtain a deviation profit of \( (N-1)\Pi^M/N \) in the same way as before, by offering each unintegrated downstream firm the contract \( (w^M - e, F^M - e) \). This reduction in an upstream firm’s deviation profit is the (by now) familiar outlets effect of vertical integration. However, when downstream prices are set after observing upstream firms’ contract offers, the integrated \( D_1 \) can change its retail price \( p_1 \) (or quantity \( q_1 \)) in response to \( U_i \)’s deviation. Anticipating this reaction, \( U_i \) may choose to make a different deviation, which exploits the price flexibility of the unintegrated downstream firms and induces different behavior from them. The net effect of this price (quantity) adjustment by integrated and unintegrated downstream firms is to reduce the deviation profit of the unintegrated \( U_i \) beyond what is due to the outlets effect alone. We call this additional impact the reaction effect of vertical integration. As before, an unintegrated upstream firm makes zero profit in the punishment phase. Hence, its incentive constraint is given by

\[
\frac{(1 - \alpha)\Pi^M}{(M - 1)(1 - \delta)} \geq \Pi^M - \frac{1}{N}\Pi^M \text{ outlets effect} \\
- \left[ \left( \frac{N - 1}{N} \right) \Pi^M - \pi_{\text{dev}}^{\text{unint}} \right] \text{ reaction effect} \\
= \pi_{\text{dev}}^{\text{unint}},
\]

where \( \pi_{\text{dev}}^{\text{unint}} \) denotes the maximum deviation profit of an unintegrated upstream firm. The following lemma shows that the reaction effect of vertical integration does indeed reduce an unintegrated upstream firm’s deviation profit.

**LEMMA 1**: An unintegrated upstream firm’s maximum deviation profit, \( \pi_{\text{dev}}^{\text{unint}} \), satisfies \( \pi_{\text{dev}}^{\text{unint}} < (N-1)\Pi^M/N \).

The lemma follows from two observations. First, since the integrated downstream firm \( D_1 \) faces an effective wholesale price of zero (the marginal cost of its upstream division \( U_1 \)), \( D_1 \) must receive a share of at least \( 1/N \) of industry profits when an unintegrated upstream firm \( U_i \) deviates, and the deviant \( U_i \) will not be able to extract \( D_1 \)’s profit. Second, the industry profit when \( U_i \) deviates will be less than the monopoly profit since the integrated \( D_1 \) will choose its myopic best response to the other retail prices/quantities.

Turning to the integrated \( U_i-D_1 \)’s incentives to deviate, it can no longer obtain the business of the unintegrated downstream firms simply by slightly undercutting its upstream rivals’ contract offers (unless final goods are homogeneous and downstream competition is in prices). The reason is as follows. After it has made deviant offers to the unintegrated downstream firms, \( U_i-D_1 \) will at the downstream pricing stage set the price \( p_1 \) (or quantity \( q_1 \)) that maximizes the integrated \( U_i-D_1 \)’s deviation profit. To the extent that the wholesale price offered by \( U_i-D_1 \) to the unintegrated downstream firms is less than the monopoly price \( p^M \) (which it must be if goods are differentiated or competition is in quantities), the integrated firm’s downstream unit will optimally set a retail price below \( p^M \) (or a quantity above \( q^M \)). Of course, this will be anticipated by the unintegrated downstream firms, which will therefore reject any offer with a “high” fixed fee. Indeed, as we will show below, the integrated firm’s maximum deviation profit, \( \pi_{\text{dev}}^{\text{int}} \), will be less than the monopoly profit \( \Pi^M \) if final goods are differentiated. We refer to this reduction in \( U_i \)’s deviation profit—as a result of its vertical merger with \( D_1 \)—as the lack-of-commitment effect of vertical integration: it is the integrated firm’s inability to commit to its own retail price that reduces its deviation profit.
LEMMA 2: The integrated firm’s deviation profit $\pi_{d}^{\text{dev}}$ satisfies $\pi_{1}^{d} \leq \Pi^{d}$, where the inequality is strict if final goods are differentiated.

As in the model with simultaneous moves, there is a countervailing punishment effect since the integrated firm will be able to capture its downstream affiliate’s profit in the punishment phase, $\pi^{NC}$. Hence, the integrated $U_{1}-D_{1}$’s incentive constraint can be written as

$$\frac{\alpha \Pi^{M}}{1 - \delta} \geq \Pi^{M} - \left[ \Pi^{M} - \pi_{d}^{\text{dev}} \right] \frac{1}{1 - \delta}.$$  \hspace{1cm} \text{(10)}

Combining the upstream firm’s incentive constraints (9) and (10), we obtain the critical discount factor under (single) vertical integration: \(^{29}\)

$$\delta_{\text{seq}}^{NI} = \frac{(M - 1) \pi_{\text{unint}}^{d} + \pi_{d}^{\text{dev}} - \Pi^{M}}{(M - 1) \pi_{\text{unint}}^{d} + \pi_{d}^{\text{dev}} - \pi^{NC}}.$$  \hspace{1cm} \text{(11)}

Since we have shown that the impact of the reaction effect on unintegrated firms’ deviation profits is strictly negative, and that of the lack-of-commitment effect on the integrated firm’s deviation profits is weakly negative (strictly when goods are differentiated), it should not be surprising that our main result from Section III—that vertical integration facilitates upstream collusion—is robust to allowing downstream firms to condition their retail prices/quantities on upstream firms’ contract offers.

PROPOSITION 3: In the model with sequential moves, (single) vertical integration facilitates upstream collusion, $\delta_{\text{seq}}^{SI} < \delta_{\text{seq}}^{NI}$.

From Lemmas 1 and 2 it also follows that the sequential-move structure facilitates collusion when one firm is vertically integrated, $\delta_{\text{seq}}^{SI} < \delta_{\text{seq}}^{NI}$ (note that $\delta_{\text{seq}}^{NI} = \delta_{\text{seq}}^{NI}$). We can interpret this result as suggesting that if the industry contains one integrated firm, it will be helpful to collusion if upstream prices are relatively sluggish in the sense that downstream firms can change their prices (or quantities) more quickly than upstream firms can.

V. Conclusion

In this paper we show how vertical mergers can facilitate upstream collusion. Two important effects are relevant to the analysis: the outlets effect and the punishment effect. The outlets effect arises because upstream firms cannot profitably sell through the downstream outlets owned by their integrated upstream rivals when they choose to deviate; this effect reduces the profitability of deviation and hence facilitates collusion. The punishment effect, on the other hand, arises because an integrated firm cannot be punished as severely as an unintegrated one when downstream firms make rents in the punishment phase; this effect makes sustaining collusion more difficult in the presence of an integrated firm. Our main result is that, whether downstream firms compete in prices or quantities to sell differentiated or homogeneous goods, the outlets effect always dominates the punishment effect in an unintegrated industry, so that the first vertical merger always facilitates collusion.

We demonstrate that our result is robust to upstream firms making secret contract offers, and to the use of an optimal punishment scheme as opposed to Nash reversion (see the Web Appendix). We also consider an alternative timing for our model where downstream prices or quantities are set after upstream offers have been made (as opposed to simultaneously, as in the baseline model). In this case, downstream firms’ ability to adjust their downstream strategic variable in response to out-of-equilibrium upstream offers results in two further effects: the reaction effect and the lack-of-commitment effect. The reaction effect arises because integrated firms can respond aggressively in downstream markets to the upstream deviations of their rivals, reducing the latter’s profits from deviation. The lack-of-commitment effect arises because integrated firms can also best respond downstream to their own deviant upstream offers: a response that

\(^{29}\) As in the baseline model, when final goods are homogeneous and downstream competition is in prices, there exists an equilibrium in which upstream collusion can be sustained for any discount factor under vertical integration. See footnote 18.
is anticipated by the unintegrated downstream recipients of such offers and reduces their willingness to pay for deviant contracts. Both effects reduce firms’ deviation profits and hence make collusion easier to sustain, further reinforcing our main result.

In our baseline model, we also examine the effect of multiple vertical mergers. In general, the net effect on collusive possibilities of multiple vertical mergers is ambiguous, but can in principle be calculated given a demand function and a market structure. We provide conditions under which every vertical merger in an industry facilitates collusion. Thus, our analysis has potentially important implications for the antitrust policy pertaining to vertical mergers.

APPENDIX

PROOF OF PROPOSITION 2:
Part (i): To see this, note that
\[
\lim_{N \to \infty} N \left[ \frac{M - 1}{M} - \delta(K) \right] = \frac{(M - 1)K}{M^2}
\]
is positive and strictly increasing in \(K\). Hence, \(\delta(K) < \delta(K - 1)\) for \(K \geq 1\) and sufficiently large \(N\).

Part (ii): This follows from two observations. First, \(\delta(K) \to 1\) as \(M \to \infty\) if \(K < N\). Second, \(\delta(N) = [N\pi_{\text{int}}^{\\text{dev}}(N) - \Pi^M]/[N\pi_{\text{int}}(N) - N\pi^{\\text{NC}}] < 1\), independently of \(M\).

Part (iii): We have \(\tilde{\delta}(K) < \tilde{\delta}(K - 1)\) if and only if
\[
\left\{- (M_s + N_s - 2K) + N_s \left[ \frac{\pi_{\text{int}}^{\\text{dev}}(K)}{\Pi^M} - (K - 1) \frac{\pi_{\text{int}}^{\\text{dev}}(K - 1)}{\Pi^M} \right] \right\} \left\{ N_s - N_s(K - 1) \frac{\pi^{\\text{NC}}}{\Pi^M} \right\}
\]
\[
< N_s \frac{\pi^{\\text{NC}}}{\Pi^M} \left\{ (M_s - K + 1)(N_s - K + 1) + N_s \left[ (K - 1) \frac{\pi_{\text{int}}^{\\text{dev}}(K - 1)}{\Pi^M} - 1 \right] \right\}.
\]

Under our assumptions, the left-hand side converges to \(-\mu\) as \(s \to \infty\), while the right-hand side converges to 0.

PROOF OF LEMMA 1:
Consider a deviation by the unintegrated upstream firm \(U_i, i \geq 2\). Suppose first that \(U_i\) deviates by offering contracts of the form \((0, F_{ij})\) to each of the \(N - 1\) unintegrated downstream firms, and that the fixed fee \(F_{ij}\) is low enough that each downstream firm will accept \(U_i\)’s offer. Then, at the downstream pricing stage, each downstream firm will set a price of \(p^{\\text{NC}}\) (or a quantity of \(q^{\\text{NC}}\), and so the integrated \(D_1\)’s profit will be \(\pi^{\\text{NC}}\), while the deviant \(U_i\) can extract at most \((N - 1)\pi^{\\text{NC}} < (N - 1)\Pi^M/N\) from the \(N - 1\) unintegrated downstream firms. Hence, in this case, \(U_i\)’s deviation profit is indeed less than \((N - 1)\Pi^M/N\).

Suppose now that \(U_i\) deviates by offering contracts that involve a positive wholesale price, \(w_{ij} > 0\), for at least one downstream firm \(D_j\) (or a high fixed fee \(F_{ij}\) so that at least one unintegrated downstream \(D_j\) will reject the deviant offer). In this case, the integrated \(D_1\) will (in that period) obtain a share of the industry profit that strictly exceeds \(1/N\) (since \(D_1\) faces an effective wholesale price of zero, while either at least one of its rivals faces a higher wholesale price or else at most \(N - 2\) unintegrated downstream firms are active), which the deviant \(U_i\) will not be able to extract. Since the industry profit is bounded from above by the monopoly profit, this means that the deviant \(U_i\) can extract strictly less than \((N - 1)\Pi^M/N\) from the \(N - 1\) unintegrated downstream firms.
PROOF OF LEMMA 2:
The inequality $\pi_{\text{dev}} \leq \Pi^M$ holds trivially: if $\pi_{\text{dev}} > \Pi^M$, at least one downstream firm would make a loss and thus would have a profitable deviation. We now show that $\pi_{\text{dev}} < \Pi^M$ if final goods are differentiated. To see this, note that for the deviant upstream firm to extract all of the monopoly profits when final goods are differentiated (by downstream firm), it would need to sell through all of the $N$ downstream firms, and each of the downstream firms would need to set the price $p^M$ (or quantity $q^M = Q^M/N$). Suppose now that the integrated $U_1-D_1$ offers contracts of the form $(w_i',F_i')$ to each unintegrated downstream firm $D_j$, $j \geq 2$. Suppose further that all unintegrated downstream firms accept and set the monopoly price $p^M$ (or quantity $q^M$). Because of double marginalization (which must arise when products are differentiated), the wholesale price $w_i'$ must be less than the monopoly price, $w_i' < p^M$. At the downstream pricing stage, the integrated $U_1-D_1$ would then face the following optimization problem:

$$\max_{p_i} \Sigma_{p_i} Q(p_i;p^M,...,p^M) + \Sigma_{j \geq 2} w_i' Q(p_i;p^M,...,p^M).$$

Since $w_i' < p^M$, the integrated $U_1-D_1$ would thus optimally set a retail price $p_i < p^M$. But this means that the integrated firm cannot extract all of the monopoly profits. (The same argument applies when downstream competition is in quantities rather than prices.)

PROOF OF PROPOSITION 3:
Vertical integration facilitates upstream collusion, $\delta_{\text{seq}}^{SI} < \delta_{\text{seq}}^{NI}$ if

$$\begin{align} 
\max_{\pi_i} \Sigma_{\pi_i} + [\pi_{\text{dev}} + \pi_{\text{NC}}] &< \Pi^M.
\end{align}$$

Lemma 1 implies that $\pi_{\text{dev}} + \pi_{\text{NC}} < \Pi^M$, while Lemma 2 states that $\pi_{\text{dev}} \leq \Pi^M$. Hence, equation (12) must hold, and so $\delta_{\text{seq}}^{SI} < \delta_{\text{seq}}^{NI}$.

REFERENCES


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