Concentration Thresholds for Horizontal Mergers*

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Abstract

Concentration-based thresholds for horizontal mergers, such as those employed in the U.S. DOJ and FTC Horizontal Merger Guidelines, play a central role in merger analysis. However, the basis for these thresholds, in both form and level, remains unclear. We show that there is both a theoretical and an empirical basis for focusing solely on the change in the Herfindahl index, and ignoring its level, in screening mergers for whether their unilateral price effects will harm consumers. We also argue, again both theoretically and empirically, that the levels of current thresholds likely are too lax, unless one expects efficiency gains of 5% or greater from the typical merger, or other factors such as entry and product repositioning to significantly constrain the exercise of market power post-merger.

1 Introduction

Concentration measures play a central role in merger analysis. The current U.S. Department of Justice (DOJ) and Federal Trade Commission (FTC) Horizontal Merger Guidelines state various presumptions – both safe harbor presumptions and presumptions of anticompetitive effects – based on the level of the post-merger Herfindahl index and the change that the

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merger induces in that index (both naively computed, by adding the merging firms’ pre-merger shares together). While many other factors come into play in the agencies’ analyses, these concentration-based presumptions have a significant impact on agency decisions, both in screening mergers for further review and in ultimately deciding whether to challenge them. At trial, the “structural presumption” that arises when a merger exceeds the Guidelines’ threshold for likely anticompetitive effects heavily influences court decisions. Surprisingly, perhaps, the basis for these presumptions in both form and level remains unclear.

In this paper, we examine these presumptions, focusing on a merger’s likely unilateral price effects. We make two points: First, we show that there is both a theoretical and an empirical basis for focusing solely on the change in the Herfindahl index, and ignoring its level, in screening mergers for whether their unilateral effects will harm consumers. This point has been recognized by others before us (e.g., Shapiro, 2010; Froeb and Werden, 1998) and, indeed, is made in the 2010 Guidelines for the case of mergers in differentiated product industries, but is still not yet widely appreciated. Here we go further in demonstrating why this is so theoretically and in providing empirical evidence in support of this proposition. Second, we argue, again both theoretically and empirically, that the levels at which the presumptions currently are set likely are too lax, at least unless one is crediting large synergies (5% or greater) or a significant presumption that entry, repositioning, or other factors would prevent any anti-competitive effects of the typical merger.

The reliance on concentration thresholds to evaluate unilateral effects of mergers can make sense when information on margins, diversion ratios and cost synergies are unknown (e.g., at an initial screening stage) or of less than certain reliability (e.g., even at trial).

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1 Throughout the remainder of the paper, when we refer to the post-merger “Herfindahl index” or the “change in the Herfindahl index,” we mean the naively-computed post-merger level and change, respectively.
2 At the initial stage of deciding whether to issue a second request, the impact of the Guidelines’ thresholds may be more directional, as even precise market shares will often be unknown. Still, a rough sense of shares in the possible relevant markets that might be asserted at trial can influence these decisions since, as Shapiro and Shelanski (2021) note, “The typical route to victory for the government has been to define the relevant market and show that the merger significantly increases concentration in that market....”
3 As Hovenkamp and Shapiro (2018, p. 1997) observe, “While the technical analysis and the size of the relevant numbers have shifted somewhat over time, the basic structural presumption and burden shifting framework remain alive and well.” Shapiro and Shelanski (2021) remark that “in almost every case where the government establishes the structural presumption, the government wins” and that “[t]he structural presumption remains the central route by which the government wins merger challenges in court.” Peters and Wilder (2021) note that “In all six of the litigated Division horizontal merger cases that yielded a judicial opinion, the opinions directly cited the 2010 HMG concentration thresholds.”
4 See Schmalensee (1987, pp. 47-50) for one previous discussion of the Guidelines concentration thresholds.
5 Miller and Weinberg (2017) provide evidence that horizontal mergers may also lead to coordinated effects, as noted in the agencies’ Guidelines.
6 Shapiro (2010, p. 63, fn 53) notes that “There is no good link between the level of the HHI and unilateral price effects with differentiated products.” See also his discussion on pp. 68-9. The 2006 Commentary on the 1992 Guidelines makes a similar point on p.16, noting that for unilateral effects “[t]he concentration of the remainder of the market often has little impact on the answer....”
7 Kwoka (2017) reaches a similar conclusion about the safe harbor in a study examining outcomes identified in retrospective studies of 9 mergers.
8 Another argument sometimes made for a concentration-based standard is the greater certainty it provides
We therefore adopt the perspective that there is some standard presumption regarding cost synergies and elasticities that an agency or court implicitly uses when evaluating whether the concentrating effects of a horizontal merger are of concern. As we discuss later, sensible presumptions regarding elasticities may depend on how narrowly markets are being defined.

The paper is organized as follows. In Section 2, we review the history of concentration-based thresholds in the various versions of the *Horizontal Merger Guidelines* and discuss the interaction of market definition with these thresholds.\(^9\)

In Section 3, we examine three canonical models of competition in which one might hope that there would be a clear relationship between equilibrium concentration measures and the effect of a merger on consumer surplus: the Cournot model of output/capacity competition in homogeneous good industries, and the multinomial logit and constant elasticity of substitution models of differentiated product price competition. As in Werden (1996), Froeb and Werden (1998), and Farrell and Shapiro (2010), our focus in this analysis is on the level of marginal cost reduction (the “synergy” or “efficiency gain”) required to prevent a merger from harming consumers.\(^10\) We show that this critical level of efficiencies depends in these models on the merging firms’ shares, but not on the shares of non-merging firms. In fact, for mergers between symmetric firms in the Cournot model, given the market demand elasticity, the required synergy depends solely on the change in the Herfindahl index, and not at all on its post-merger level. We also examine how the levels of the required synergies depend on the merging firms’ shares. In the Cournot model, with synergies of 3% and common levels of market demand elasticity, consumer harm occurs when the merging firms’ shares are much like those in the 1968 *Guidelines*’ thresholds. In contrast, the threshold levels of merger-induced change in the Herfindahl index are more lenient, but still restrictive, in the multinomial logit and constant elasticity of substitution models of price competition.

The theoretical models of Section 3 are certainly special. In Section 4, we provide an empirical investigation of how mergers’ effects on consumers are related to concentration measures in one industry. We focus on possible mergers in brewing. Using the estimated demand system for leading beer brands in Miller and Weinberg (2017), a random-coefficient nested logit demand system that is not covered by our theoretical analysis, and treating each local market separately, we compute for various hypothetical (local) mergers the efficiency improvement that would be required to prevent consumer harm. The results show that, as in the models of Section 3, the required efficiency gain is strongly related to the change in the Herfindahl index and not very related to the level of the post-merger Herfindahl (once

\(^9\)Throughout, we focus on the U.S. agencies’ concentration-based thresholds, but similar points apply to many other jurisdictions. For example, the European Commission also has horizontal merger guidelines that adopt thresholds based on the Herfindahl level and its change. Also, throughout we will use “concentration screens” and “concentration-based thresholds” interchangeably, referring to their use both in the initial screening stage as well as in any later litigation.

\(^10\)Our analysis is thus complementary to that in Nocke and Schutz (2019) who show that, absent efficiencies, the merger-induced loss in consumer surplus is approximately proportional to the change in the Herfindahl index, where the approximations are taken around small market shares and around monopolistic competition conduct.
The levels of the merger-induced change in the Herfindahl necessary to prevent consumer harm in these local beer markets generally fall in the range of those we derive in the theoretical models of Section 3: if the typical merger in these markets would result in a 3% efficiency gain then consumer harm arises once the merger-induced change in the Herfindahl index exceeds a threshold somewhere between 150-200. In this case, for a simple rule that approves mergers below a threshold change in the Herfindahl and rejects them above that level, the consumer surplus-maximizing threshold is between 113 and 181, depending on the version of the Miller-Weinberg estimated demand system and the form of the market shares (volume-based or revenue-based) that we employ. We show as well that for these mergers, this simple policy generates a level of consumer surplus gain close to the optimum. Moreover, it performs much better than both the 1982 and 2010 Guidelines’ thresholds, which do not even produce a positive change in consumer welfare until efficiency gains reach 5%. Finally, we show that for efficiency gains below 5%, the 1982 Guidelines’ thresholds out-perform those of the 2010 Guidelines, and the 1968 Guidelines out-perform the 1982 Guidelines if efficiency gains are below 2-3%.

In Section 5 we provide a discussion of our results. We first discuss a number of factors that our analysis ignores that could possibly make the level of the Herfindahl play a useful role in screening mergers. While we do not discount any of these possibilities, we view our results as raising the bar for the level of theoretical and empirical support that should back up any such claim. Second, we discuss the extent to which our results support the view that current thresholds likely are too lax. Here we observe that the literature on efficiency effects of horizontal mergers is extremely limited, and discuss its findings. Ultimately, while we view 5% or greater efficiency gains as unlikely in the typical merger, more reliable evidence on typical efficiency gains for mergers near screening thresholds would clearly be beneficial.

We conclude in Section 6.

2 History of the Merger Guideline Concentration Screens

The first version of the Merger Guidelines – issued solely by the Department of Justice – appeared in 1968, shortly after the 1963 Philadelphia National Bank decision and roughly contemporaneous with the Neal Report on antitrust policy. As described by Shapiro (2010), the 1968 Guidelines approach toward horizontal mergers was focused entirely on preventing increases in concentration and it proposed concentration thresholds that were markedly more stringent than those today. Those presumptions, summarized in Figure 1, were largely dependent on the shares of the two merging firms. For mergers in markets in which the four-firm concentration ratio was above 75%, a merger would be blocked if a firm with a 4% share wanted to acquire another firm with a 4% share, and a firm with a 15% share could not acquire a firm with a 1% share.\footnote{Somewhat curiously, the 1968 screens depended on which merger partner was the acquirer.} For markets with a four-firm concentration ratio below
75%, the thresholds were not much more lenient: a merger between two 5% firms would be blocked.

The DOJ’s 1982 *Guidelines* represented a marked change, with the Herfindahl index (HHI) replacing the four-firm concentration ratio, but more importantly with the level of market concentration having much more importance, and with much more lenient standards.\(^{12}\)\(^{13}\) Figure 2 depicts the 1982 screening thresholds, which depend on the post-merger level of the HHI (measured out of 10,000) and the merger-induced change in HHI (labelled “\(\Delta\)HHI” in the figure). For example, a merger between two 5% share firms, which would lead to a 50 point increase in the HHI, rather than being challenged became presumptively legal. More specifically, mergers in “unconcentrated” markets with a post-merger HHI below 1000 fell into the (moderately shaded) green zone of the figure, representing mergers that were unlikely to be challenged. In “moderately concentrated” markets, with post-merger Herfindahl indices between 1000 and 1800, a merger was “more likely than not” to be challenged if it fell into the (lightly shaded) yellow zone because its \(\Delta\)HHI was above 100, while mergers for which \(\Delta\)HHI was below 100 fell into the green zone. In “highly concentrated markets” with a post-merger HHI above 1800, mergers whose \(\Delta\)HHI were below 50 fell into the green zone, those with \(\Delta\)HHI between 50 and 100 fell into the yellow zone, while those with \(\Delta\)HHI above 100 fell into the (darkly shaded) red zone that meant that the DOJ was “likely to challenge.” The 1992 *Horizontal Merger Guidelines*, issued for the first time jointly by the DOJ and FTC, maintained these presumptions.\(^{14}\)

\(^{12}\) Carlton and Peltzman (2010) attribute the shift to use of the HHI to then-Assistant Attorney General William Baxter’s admiration for George Stigler’s 1964 “Theory of Oligopoly” paper in which Stigler (1964) related the likelihood of effective collusion to the Herfindahl index. The DOJ and FTC *Guidelines* have continued to use the HHI even as most horizontal merger cases have come to emphasize unilateral effects.

\(^{13}\) Shapiro (2010) describes well the other significant innovations in the 1982 *Guidelines*, and the continuing increase over time in consideration of other market factors in analyzing prospective mergers. One factor that may have ameliorated to some extent the more lenient standards was the introduction in the 1982 *Guidelines* of the “hypothetical monopolist test” for market definition, which may have led to narrower market definitions.

\(^{14}\) The 1992 *Guidelines* did change “more likely to be challenged than not” for the yellow zone to “potentially raise significant competitive concerns.” The 1982 *Guidelines* also had a presumption of anticompetitive harm where the acquirer was the leading firm in the industry, had a share of at least 35%, was more than twice as large as the second largest firm, and was acquiring a firm with at least a 1% share. This presumption was eliminated in the 1992 *Guidelines*. 

**Figure 1:** Anticompetitive presumptions in the 1968 *Merger Guidelines*
Most recently, the 2010 revision of the Horizontal Merger Guidelines relaxed these standards. As depicted in Figure 3, it raised the safe harbor level of the HHI from 1000 to 1500, raised the threshold for considering a market highly concentrated from 1800 to 2500, and raised the critical levels of $\Delta HH$ in highly concentrated markets from 50 to 100 for the safe harbor, and from 100 to 200 for the presumption of harm ($\Delta HH$ thresholds in moderately concentrated markets were not changed).\(^{15}\)

Notably, while the theoretical and empirical basis for neither the 1968 Guidelines concentration thresholds nor the 1982 changes were ever clearly laid out by the agencies, the reason for the change in 2010 was made explicit: the aim was to enhance transparency by making the thresholds conform more closely with actual agency practice (see Shapiro, 2010). So, once again, no explicit economic rationale was offered.

Figure 4 depicts actual FTC enforcement results for those horizontal mergers that received second requests from 1996-2011.\(^{16}\) A merger received an “enforcement action” if the FTC sought to block or modify it. Evident in the figure is both the strong effect of the level of concentration on the likelihood of enforcement and the fact that many mergers that fell into the “red zone” anticompetitive presumption of the 1992 (and even the 2010) Guidelines nonetheless were approved in the end without conditions. (Zero bars in the figure often represent cases in which there were no mergers reviewed in that category.) Table 7 in Appendix A gives the statistics underlying Figure 4. Also notable in those statistics is the fact that of the 1359 second requests considered in Figure 4, only 29 involved mergers with $\Delta HH$ less than 100, 114 had a $\Delta HH$ less than 200, and 210 had a $\Delta HH$ less than 300. Since many notified mergers are likely to have such levels of $\Delta HH$, it appears to be highly likely that such mergers are simply allowed without further scrutiny.

\(^{15}\)At the same time, the 2010 revision continued the move of the Guidelines away from rigid structural presumptions and toward reliance on a range of evidence of potential anticompetitive effects in making final determinations about whether to initiate an enforcement action.

**Figure 3:** Screening thresholds in the 2010 *Horizontal Merger Guidelines*

![Graph showing screening thresholds in the 2010 Horizontal Merger Guidelines](image)

**Figure 4:** FTC horizontal merger enforcement frequencies for mergers receiving a second request, as a function of the post-merger level of the Herfindahl index and the merger-induced change in the Herfindahl index, 1996-2011. [Source: Federal Trade Commission (2013)]

![Bar chart showing FTC horizontal merger enforcement frequencies](image)
Beginning with the 1982 version, the Guidelines’ have specified a procedure, the “hypothetical monopolist test,” for determining relevant markets, and hence levels of concentration. The test identifies a collection of products as a relevant market if a hypothetical monopolist of these products would “likely impose at least a small but significant and non-transitory increase in price (‘SSNIP’) on at least one product in the market, including at least one product sold by one of the merging firms.” In practice, this increase has been taken to be five percent.

As we discuss in Appendix D, the test can often support quite narrow relevant markets, sometimes including just the products of the merging firms. The 2010 Guidelines state that “the agencies may evaluate a merger in any relevant market satisfying the test” but that when measuring concentration the agencies “usually do so in the smallest market satisfying the hypothetical monopolist test.” In practice, however, this does not appear what the agencies actually do in litigation: when the agencies challenge a merger they typically assert broader more ‘natural’ markets that maintain the structural presumption, likely due to a concern that courts may look dimly on what could appear to be overly narrow markets. If these more natural market definitions are necessary to win in court (or reach suitable settlements given the threat of going to court), then concentration measures in such markets become what is relevant for the question of whether merger policy is too lax. When we look at elasticities in our theoretical analysis, or measure concentration in our empirical analysis, we will do so with such markets in mind.

3 Theoretical Analysis

Analysis of horizontal mergers focuses on weighing the risk of anticompetitive reductions in competition against the prospect for merger-related efficiencies. Concentration screens for mergers must therefore aim to capture, based on firms’ market shares, the likely balance of these two effects for the “typical” merger. Since absent any efficiency gains a horizontal merger will generally (weakly) increase prices, any merger screen aimed at preventing consumer harm that would allow some mergers and block others must implicitly be relying on some presumption of the efficiency gain (the “synergies”) that should be credited to a typical merger. As such, we focus throughout the paper on how the required efficiency gain is related to measures of concentration.

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17 Horizontal Merger Guidelines (2010, p. 9).
19 For example, since 2008 the DOJ has challenged at least four mergers involving Anheiser Busch in the beer industry that we analyze in Section 4. In each case the DOJ complaint alleged a market for “beer,” despite the fact that (according to the demand estimates we rely on in Section 4) in each case a much narrower market definition would likely have satisfied the hypothetical monopolist test. Likewise, in the 2017 Aetna-Humana merger, the DOJ alleged a market for Medicare Advantage plans. Yet, the huge price increases in the merger simulation DOJ’s expert Aviv Nevo presented at trial make clear that the merging firms’ Medicare Advantage plans alone would have passed the hypothetical monopolist test (see Bayot et al., 2019).
In general, models of oligopolistic competition need not produce a clean relationship between the effect of a merger and market shares, let alone concentration measures such as the Herfindahl index. In this section, however, we focus theoretically on three models that do: the Cournot model of output/capacity competition in a homogeneous good industry and the multinomial logit and constant elasticity of substitution models of price competition.20

3.1 Mergers in the Homogeneous-Goods Cournot Model

Consider an industry with a set \( F \) of firms producing a homogeneous good with constant returns to scale and competing in a Cournot fashion. Let \( c_f \) denote the (constant) marginal cost of firm \( f \in F \), and \( P(Q) \) inverse demand, where \( Q \) is aggregate output. We impose standard assumptions ensuring that there exists a unique Nash equilibrium in quantities: that for any \( Q \) such that \( P(Q) > 0 \), we have \( P'(Q) < 0 \) and \( P'(Q) + QP''(Q) < 0 \); moreover, \( \lim_{Q \to \infty} P(Q) = 0 \).

Let \( Q^* \) denote the pre-merger aggregate equilibrium output. For simplicity, we assume that all firms in \( F \) are active before the merger in that \( \max_{f \in F} c_f < P(Q^*) \). The pre-merger market share of firm \( f \), \( s_f \), satisfies

\[
s_f = -\frac{P(Q^*) - c_f}{Q^*P'(Q^*)},
\]

and the pre-merger Herfindahl index is given by \( H = \sum_{f \in F} s_f^2 \).

Consider a merger \( M = \{m, n\} \) between firms \( m \) and \( n \). Given their pre-merger market shares \( s_m \) and \( s_n \), their combined pre-merger market share is \( s_M \equiv s_m + s_n \). The post-merger Herfindahl index is given by

\[
\overline{H} = s_M^2 + \sum_{f \in F \setminus M} s_f^2,
\]

and the merger-induced change in the index by \( \Delta H \equiv \overline{H} - H = 2s_ms_n \). For reasons that will become clear later, let \( H_M \equiv (s_m^2 + s_n^2)/s_M^2 \) denote the within-merger Herfindahl index, whose value lies between 1/2 and 1, and let

\[
c_M = \frac{s_mc_m + s_nc_n}{s_M}.
\]

denote the output-weighted average marginal costs of the merger partners prior to the merger. We denote the merged firm’s post-merger marginal cost by \( \overline{c}_M \).

We seek to relate the merger-induced efficiency gains necessary to make the merger have no effect on consumer surplus — that is, to be “CS-neutral” — to the pre-merger market structure.21 Recall from Farrell and Shapiro (1990) (see also Nocke and Whinston, 2010)

20 The multinomial logit and constant elasticity of substitution models of price competition both have the IIA property in which diversion is proportional to market shares (measured in volume in the multinomial logit model and in revenue in the constant elasticity of substitution model).

21 Under the regularity conditions we assume, a reduction in a firm’s marginal cost expands output and
that merger $M$ is CS-neutral if and only if

$$P(Q^*) - \bar{c}_M = [P(Q^*) - c_m] + [P(Q^*) - c_n].$$

(1)

It is instructive to begin with the simple case in which the two merger partners are symmetric: $c_m = c_n \equiv c_M$ and thus $s_m = s_n \equiv s_M/2$. Using equation (1), the merger is CS-neutral if the fractional change in the merger partners’ marginal cost satisfies

$$\frac{c_M - \bar{c}_M}{c_M} = \frac{P(Q^*) - c_M}{c_M}.$$

(2)

From the merger partners’ pre-merger first-order conditions, we have

$$c_M = P(Q^*) \left[1 - \frac{s_M}{2\epsilon}\right],$$

where $\epsilon \equiv -P(Q^*)/[Q^* P'(Q^*)]$ is the pre-merger price elasticity of demand. Substituting for $c_M$ on the right-hand side of equation (2), we obtain

$$\frac{c_M - \bar{c}_M}{c_M} = \frac{s_M}{2\epsilon} = \frac{\Delta H}{2} \epsilon - \frac{\Delta H}{2}.$$

(3)

That is, for a given demand elasticity, the required efficiencies are perfectly related to and increasing in the change in the Herfindahl index, and completely independent of the level of the Herfindahl index. Any relationship between consumer harm and the level of the Herfindahl index would therefore need to come through a relationship between the Herfindahl and the elasticity of demand, but in general there is no clear theoretical relationship between the two.\(^{22}\)

The change in the Herfindahl required to prevent harm to consumers at various levels of the market demand elasticity and efficiency gain are also striking. Table 1 shows these levels, as well as the corresponding market share levels for each of the (symmetric) merging firms. For example, in a market with a demand elasticity of 1.5, a merger of symmetric firms that results in a 5% synergy would lower consumer surplus if the change in the Herfindahl exceeds 102, which corresponds to each of the merging firms having roughly a 7% share. Were the industry symmetric, that would be a market with 14 firms. With a 3% synergy the change in the Herfindahl would need to be below 38 to prevent consumer harm, regardless of the level lowers price. Thus, any larger synergy than that required for CS-neutrality will result in the merger benefiting consumers, while any lower synergy will result in the merger harming consumers.

\(^{22}\)Under the standard regularity conditions we assume, an increase in output lowers the elasticity of demand. However, as is well known, there is no clear relationship between the level of the Herfindahl index and the level of output in a market. For example, an increase in the number of firms will raise output and lower the Herfindahl index, but a reduction in cost for the most efficient firm in the market will raise output but increase the Herfindahl index. As well, the elasticity of demand may affect the number of active firms and thus the Herfindahl index.
of the post-merger Herfindahl. This is a level similar to that in the 1968 Guidelines. Still, in markets in which the elasticity of demand reaches 2.5, with a 5% synergy some mergers that fall into the anticompetitive presumption category of the 2010 Guidelines because they have a post-merger Herfindahl above 2500 and a change in the Herfindahl above 200 would actually be beneficial for consumers (if \( \Delta H < 283 \)).

**Table 1:** Maximal Level of Individual Shares and \( \Delta H \) (*10,000) To Prevent Consumer Harm for Various Levels of Cost Synergy in the Cournot Model

<table>
<thead>
<tr>
<th>Demand Elasticity</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>7.5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>( \Delta H )</td>
<td>2</td>
<td>7</td>
<td>17</td>
<td>30</td>
<td>45</td>
<td>97</td>
<td>165</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>( \Delta H )</td>
<td>4</td>
<td>17</td>
<td>38</td>
<td>67</td>
<td>102</td>
<td>219</td>
<td>372</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>( \Delta H )</td>
<td>8</td>
<td>30</td>
<td>68</td>
<td>118</td>
<td>181</td>
<td>389</td>
<td>661</td>
</tr>
<tr>
<td>2.5</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td>( \Delta H )</td>
<td>12</td>
<td>48</td>
<td>106</td>
<td>184</td>
<td>283</td>
<td>608</td>
<td>1033</td>
</tr>
</tbody>
</table>

Importantly, Table 1 also shows that if elasticity information is available, it should be used to adjust the screening thresholds for the required change in the Herfindahl.\(^{24}\) This relation between screening thresholds and the demand elasticity also indicates how these screening thresholds should interact with market definition, since the narrower the “market,” the higher the market demand elasticity is likely to be.

Proposition 1 shows how condition (3) generalizes to the case of mergers between asymmetric firms.\(^{25,26}\)

\(^{23}\)In Section 5 we discuss what is known about typical efficiency gains and conclude that, in our view, presuming a 5% efficiency gain for a typical merger seems unduly optimistic.

\(^{24}\)Note that there is a relation between the market demand elasticity \( \epsilon \) and the pre-merger HHI given margins: \((p - c)/p = H/\epsilon\), where \( c \) is the industry-level average marginal cost (Cowling and Waterson, 1976). However, recall that we take the perspective that reliable margins are not available, so other a priori information on likely elasticities would need to be used here. Note as well that condition (3) can be derived using the difference between the pre- and post-merger conditions that \((p - c)/p = H/\epsilon\) and \((p - (1 - s - M)c - s_M\bar{\tau})/p = \overline{H}/\epsilon\).


\(^{26}\)Note that with asymmetric merger partners our notion of merger-induced efficiencies differs from the notion of “synergies” in Farrell and Shapiro (2010), which only counts reductions in marginal cost below the pre-merger marginal cost of the more efficient merger partner as a merger synergy, and not any benefits from reshuffling production across the merger partners. We believe that the measure we employ here is a
Proposition 1. For merger $M$ to be CS-neutral, the merger-induced efficiencies have to satisfy:

\[
\frac{c_M - \bar{c}_M}{c_M} = \frac{\left(\sqrt{\frac{\Delta H}{2}}\right) \left(\sqrt{2(1 - H_M)}\right)}{\epsilon - \left(\sqrt{\frac{\Delta H}{2}}\right) \left(\frac{H_M \sqrt{2}}{\sqrt{1 - H_M}}\right)}.
\]

(4)

Proof. We have

\[
\frac{c_M - \bar{c}_M}{c_M} = \frac{s_m c_m + s_n c_n - s_M \bar{c}_M}{s_m c_m + s_n c_n} = \frac{s_m c_m + s_n c_n - s_M [c_m + c_n - P(Q^*)]}{s_m c_m + s_n c_n} = \frac{s_n [P(Q^*) - c_m] + s_m [P(Q^*) - c_n]}{s_m c_m + s_n c_n} = \frac{s_m P(Q^*) \frac{s_m}{\epsilon} + s_n P(Q^*) \frac{s_n}{\epsilon}}{2 s_m s_n} = \frac{2 s_m s_n}{s_m \left[1 - \frac{s_n^2 + s_m^2}{s_m \epsilon}\right]} = \frac{\Delta H}{s_m \epsilon} \left(\sqrt{2(1 - H_M)}\right)
\]

(5)

where the first equality follows from the definition of $c_M$, the second from equation (1), the fourth from the pre-merger first-order conditions, and the last from substituting for $s_M$ using the fact that since

\[
\Delta H = (s_M)^2 (1 - H_M)
\]

we have

\[
s_M = \sqrt{\frac{\Delta H}{1 - H_M}}.
\]

Intuitively, one would expect that, holding the change in the Herfindahl index fixed, the required efficiency shrinks as the merging firms become more asymmetric. (When one of the merging firms has zero share, there is no anticompetitive effect of the merger even absent efficiency gains.) The following corollary confirms this.

more natural benchmark when thinking about the efficiency gains to be credited to a typical merger. As Farrell and Shapiro show, however, for a merger to not harm consumers in a Cournot market, it must involve synergies in their sense.
Corollary 1. In the Cournot model, the marginal cost reduction required to prevent a reduction in consumer surplus falls with a sum-preserving spread of the merging firms’ shares.

Proof. Substituting for $H_M$ in expression (5), using the fact that (6) implies that

$$H_M = 1 - \frac{\Delta H}{(s_M)^2},$$

yields

$$\frac{c_M - \bar{c}_M}{c_M} = \frac{\Delta H}{s_M(\epsilon - s_M) + \Delta H}.$$  

Holding $s_M$ fixed, the right-hand side is increasing in $\Delta H$, which reaches its maximum when the merging firms are symmetric and is monotonically decreasing as they become more asymmetric.

3.2 Mergers in Differentiated Goods Industries with Price-setting Competition

We now consider mergers between multiproduct firms offering differentiated goods and competing in prices. There is a set $\mathcal{N}$ of horizontally differentiated products offered by firms in set $\mathcal{F}$. Each product $k \in \mathcal{N}$ is offered by only one firm but each firm $f \in \mathcal{F}$ may offer multiple products, $f \subset \mathcal{N}$. As in the Cournot model analyzed above, we assume that firms have constant returns to scale, with $c^k$ denoting the marginal cost of product $k$.

We focus on two demand systems: constant elasticity of substitution (CES) and multinomial logit (MNL). Multiproduct-firm price competition with such demands shares a useful feature with the homogeneous-goods Cournot model: the game is aggregative in that each firm’s profit depends on the strategic choices of its rivals only through a one-dimensional aggregator, and consumer surplus depends only on the value of that aggregator. A difference between the two demand systems is that under CES demand total expenditure (including the outside good) is fixed, whereas under MNL demand total consumption (including the outside good) is fixed.

CES demand. We begin with the case of CES demand. The demand for product $k \in \mathcal{N}$ is given by

$$D^k(p^k; A) = \frac{b^k(p^k)^{-\sigma}}{A},$$

where

$$A \equiv \sum_{j \in \mathcal{N}} b^j(p^j)^{1-\sigma} + A^0,$$
is the value of the aggregator, $b^j$ and $p^j$ are the quality and price of product $j$, respectively, $\sigma > 1$ denotes the elasticity of substitution, and $A^0 \geq 0$ represents the outside good.\footnote{The parameter $\sigma$ equals the product-level own-price elasticity (of demand $D^j$) for a firm that takes the level of the aggregator $A$ as fixed. More generally, the own-price elasticity of a product $j$ considering also the effect on $A$ is $\epsilon^j = \sigma - (\sigma - 1)s^j$, while the aggregate elasticity for the inside goods is $\epsilon = \sigma - (\sigma - 1)(1 - s^0)$, where $s^0$ is the market share of the outside good.} Consumer surplus is $\text{CS}(A) = \log A$.

The profit of firm $f$ equals

$$\Pi_f((p^k)_{k \in f}) = \sum_{k \in f} (p^k - c^k)D^k(p^k; A),$$

and therefore depends on the price of any rival’s product $j \notin f$ only through the value of the aggregator $A$. From the first-order conditions of profit maximization, it can be shown that firm $f$ sets the same percentage markup $\mu_f > 0$ on each of its products,\footnote{The equilibrium analysis here follows Nocke and Schutz (2018).}

$$\frac{p^j - c^j}{p^j} = \mu_f \quad \forall j \in f,$$

and that firm $f$’s markup $\mu_f$ satisfies

$$\sigma \mu_f \left( 1 - \frac{\sigma - 1}{\sigma} \frac{T_f}{A} (1 - \mu_f)^{\sigma - 1} \right) = 1,$$

where

$$T_f = \sum_{k \in f} b^k (c^k)^{1-\sigma}$$

is firm $f$’s “type” (which equals the firm’s contribution to the aggregator – and thus to consumer surplus – if it were to price all of its products at marginal cost). Equation (8) has a unique solution in $\mu_f$, denoted $m(T_f/A)$. The function $m(\cdot)$ is called the markup fitting-in function. It is strictly increasing, $m'(\cdot) > 0$: Firms with higher types (larger $T$) or facing less competition (lower $A$) charge higher markups.

As total expenditure on all products, including the outside good, is fixed and equal to one (when normalizing the price of the outside good to one), the share of market revenue of product $k$ equals $s^k = p^k D^k(p^k; A)$. The revenue-based market share of firm $f$, $s_f = \sum_{k \in f} s^k$, can in turn be shown to satisfy

$$s_f = \frac{T_f}{A^*} (1 - \mu_f)^{\sigma - 1} \equiv S \left( \frac{T_f}{A} \right).$$

$S(\cdot)$ is called the market share fitting-in function; it is strictly increasing: $S'(\cdot) > 0$. Combining equations (8) and (9), we obtain a monotonic relationship between firm $f$’s markup
\( \mu_f \) and its market share \( s_f \):
\[
\sigma \mu_f = \frac{1}{1 - \left( \frac{\sigma - 1}{\sigma} \right) s_f}.
\]
(10)

The equilibrium aggregator level \( A^* \) is the unique solution in \( A \) to the market shares (including that of the outside good) adding up to unity:
\[
\sum_{f \in F} S\left( \frac{T_f}{A} \right) + \frac{A^0}{A} = 1.
\]

Consider now merger \( M \) between firms \( m \) and \( n \). The post-merger equilibrium value of the aggregator, \( \overline{A}^* \), then satisfies
\[
S\left( \frac{T_M}{\overline{A}^*} \right) + \sum_{f \notin M} S\left( \frac{T_f}{A^*} \right) + \frac{A^0}{A^*} = 1,
\]
where \( T_M \) is the merged firm’s type. (If the merged firm were to produce exactly the same product lines as the merger partners did jointly before the merger, at the same vector of marginal costs, then we would have \( T_M = T_m + T_n \).) Hence, the merger is CS-neutral with \( \overline{A}^* = A^* \), if \( T_M \) is such that
\[
S\left( \frac{T_M}{A^*} \right) = s_m + s_n.
\]
(11)

As shown in Nocke and Schutz (2019), for merger \( M \) to be CS-neutral, it must involve “type synergies” in that \( T_M > T_m + T_n \).

The following proposition indicates how large the type synergies have to be for the merger not to hurt consumers:

**Proposition 2.** With CES demand, for merger \( M = \{m, n\} \) to be CS-neutral, the merger-induced type synergies have to satisfy
\[
\frac{T_M}{T_m + T_n} = \frac{s_m \left( \sigma + \frac{s_m}{1-s_m} \right)^{\sigma-1} + s_n \left( \sigma + \frac{s_n}{1-s_n} \right)^{\sigma-1}}{s_m \left( \sigma + \frac{s_m}{1-s_m} \right)^{\sigma-1} + s_n \left( \sigma + \frac{s_n}{1-s_n} \right)^{\sigma-1}}.
\]
(12)

**Proof.** From equations (9) and (10), we obtain
\[
\frac{T_f}{A^*} = s_f (\sigma - 1)^{\sigma-1} \left( \sigma + \frac{s_f}{1-s_f} \right)^{\sigma-1}.
\]
(13)

---

29As \( S(0) = 0 \) and \( S''(\cdot) < 0 \), the market share fitting-in function is sub-additive. The result then follows from equation (11).
Hence, for merger $M$ to be CS-neutral, the post-merger type $T_M$ has to satisfy

$$
\frac{T_M}{A^*} = s_M (\sigma - 1)^{1-\sigma} \left( \sigma + \frac{s_M}{1-s_M} \right)^{\sigma-1}.
$$

(14)

Combining (13) and (14), yields equation (12).

Proposition 2 shows that, similar to the Cournot model, the magnitude of the required type synergies depends only on the pre-merger market shares of the merger partners and not on the concentration in the rest of the industry. As noted by Nocke and Schutz (2019), the proposition implies that a larger merger (i.e., an increase in $s_m$ or $s_n$ and thus in $s_M \equiv s_m + s_n$) requires larger synergies.

While the required type synergies depend on both merger partners’ shares, rather than simply on $\Delta H$, a sum-preserving spread of their market shares—which decreases $\Delta H$—does reduce the required type synergies:

**Corollary 2.** With CES demand, a sum-preserving spread of the merger partners’ pre-merger market shares (which decreases the change in the Herfindahl index) reduces the type efficiencies required to prevent consumer harm.

Proof. This follows from the convexity of $s(\sigma + s/(1-s))^{\sigma-1}$ in $s$, implying that a sum-preserving spread of $s_m$ and $s_n$ increases the denominator on the right-hand side of equation (12).

As the notion of type synergies may be unfamiliar, the following corollary relates the size of the required marginal cost synergies (measured as a percentage change in marginal cost, $\phi^j \equiv (c^j - c^j)/c^j$ for $j \in (m \cup n)$) to pre-merger market shares, assuming the merger does not affect the number and qualities of the merger partners’ products. The corollary does so for two specific vectors of marginal cost changes, the first focusing on marginal cost changes that leave all prices unchanged, whereas the second assuming equal percentage changes in marginal cost for all of the merging firms’ products.\(^{30,31}\)

**Corollary 3.** Suppose that the set of products offered—and the associated qualities—are not affected by merger $M = \{m, n\}$.

(i) With CES demand, all prices are unaffected by the merger if and only if for each merging firm $k = m, n$ the percentage change in the marginal cost of each of firm $k$’s products $j \in k$, $\phi^j$, satisfies

$$
\phi^j = -\frac{s_M - s_k}{(1-s_k)[\sigma(1-s_M) + s_M]}.
$$

\(^{30}\)More generally, there exists a continuum of vectors of marginal cost changes that leave consumer surplus unchanged.

\(^{31}\)Compared to the analyses for differentiated product price competition of Werden (1996) and Farrell and Shapiro (2010), our result expresses the required synergies in terms of market shares only, in contrast to their characterizations in terms of margins and diversion ratios and (in Werden, 1996) prices. Those papers’ results also focus only on deriving product-specific synergies that keep all prices unchanged, as in part (i) of Corollary 3.
(ii) With CES demand, if the marginal cost of each product \( j \in (m \cup n) \) changes by the same fraction \( \phi \), consumer surplus remains unchanged if and only if

\[
\phi = 1 - \left( \frac{s_M \left( \sigma + \frac{s_M}{1-s_M} \right)^{\sigma-1}}{s_m \left( \sigma + \frac{s_m}{1-s_m} \right)^{\sigma-1} + s_n \left( \sigma + \frac{s_n}{1-s_n} \right)^{\sigma-1}} \right)^{1/(1-\sigma)}. 
\]

Proof. See Appendix B.1.

According to part (i) of the corollary, for all prices to remain unchanged with CES demand, every product of each merger partner must have the same percentage reduction in marginal cost, with the required cost synergy being larger for the smaller merger partner. Part (ii) of the corollary gives the required synergies (i.e., percentage marginal cost changes) when those are the same for all products of the merging firms.

To get an idea of the magnitudes involved, consider a merger among symmetric firms (i.e., \( s_m = s_n \)). Table 2 depicts the maximal shares for each of the merging firms and the change in the Herfindahl index for a symmetric merger not to harm consumers for various synergy levels, assuming that there is no outside good (i.e., \( A^0 = 0 \)). The upper part of the table measures synergies in terms of percentage changes in the merging firms’ type (i.e., \( \left[ T_M - (T_m + T_n) \right]/(T_m + T_n) \), with \( T_m = T_n \) for the case of a symmetric merger), whereas the lower part measures synergies in terms of percentage changes in marginal cost (i.e., as \( \phi \times 100 \)). We report results for typical values of \( \sigma \) with CES demand.

Table 2 shows that, compared to the Cournot case, at these common levels of the substitution parameter \( \sigma \), the maximal shares and change in the Herfindahl index are larger with price competition and CES demand for any given cost synergy. Nevertheless, even a merger among relatively small firms would need to entail significant synergies to prevent consumer harm. For example, with \( \sigma = 5 \), a symmetric merger between two firms with a market share of 11% each (thus raising the Herfindahl index by 242) requires a 3% reduction in marginal cost (or more than a 10% increase in type) so as not to hurt consumers.

Note, however, that our definition of market share assumes that there is no outside good. If there is an outside good, with share \( s^0 \), the critical share levels recorded in Table 2 would need to be adjusted by the factor \( 1/(1 - s^0) \). The aggregate elasticity formula in footnote 27 implies that \( s^0 = (\epsilon - 1)/(\sigma - 1) \), where \( \epsilon \) is the aggregate price elasticity of the inside goods. For example, if \( \epsilon = 1.5 \) and \( \sigma = 5 \), the critical shares in Table 2 would increase by a factor of 1.14. As in the Cournot case, this dependence of screening thresholds on the level

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32In the international trade literature, where CES demand plays a prominent role, typical estimates of the elasticity of substitution are in the range from 4 to 6; see, for example, Costinot and Rodriguez-Clare (2014), Hottman, Redding, and Weinstein (2016), and Breinlich, Nocke, and Schutz (2020). Outside the trade literature, however, Foster et al. (2008) estimate constant elasticity plant-level demand functions for 11 homogeneous goods industries. Their estimates (see Table 2, p. 409) are mostly lower than 4, and range from 0.5 to 5.93. Product-level elasticities lower than in Table 2 above would tend to lead to lower concentration thresholds for a given level of cost synergy; for instance, for a cost synergy of 3%, the value of \( \Delta H \) in Table 2 would be 112.3 at \( \sigma = 3 \) and 57.7 at \( \sigma = 2 \).
Table 2: Maximal Level of Individual Shares and $\Delta H$ ($\times 10,000$) To Prevent Consumer Harm for Various Levels of Type Synergy (Upper Panel) and Cost Synergy (Lower Panel) with CES Demand

<table>
<thead>
<tr>
<th></th>
<th>Type Synergy:</th>
<th></th>
<th></th>
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<th></th>
<th></th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>1%</td>
<td>2%</td>
<td>3%</td>
<td>4%</td>
<td>5%</td>
<td>7.5%</td>
<td>10%</td>
</tr>
<tr>
<td>4</td>
<td>Individual shares</td>
<td>1.3</td>
<td>2.5</td>
<td>3.6</td>
<td>4.6</td>
<td>5.6</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta H$</td>
<td>3.3</td>
<td>12.2</td>
<td>25.6</td>
<td>42.6</td>
<td>62.3</td>
<td>120.6</td>
</tr>
<tr>
<td>5</td>
<td>Individual shares</td>
<td>1.2</td>
<td>2.3</td>
<td>3.4</td>
<td>4.3</td>
<td>5.3</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td>$\Delta H$</td>
<td>2.9</td>
<td>10.8</td>
<td>22.7</td>
<td>37.7</td>
<td>55.3</td>
<td>107.4</td>
</tr>
<tr>
<td>6</td>
<td>Individual shares</td>
<td>1.2</td>
<td>2.2</td>
<td>3.2</td>
<td>4.2</td>
<td>5.1</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>$\Delta H$</td>
<td>2.7</td>
<td>10.0</td>
<td>21.0</td>
<td>34.9</td>
<td>51.3</td>
<td>99.8</td>
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<table>
<thead>
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<th></th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>1%</td>
<td>2%</td>
<td>3%</td>
<td>4%</td>
<td>5%</td>
<td>7.5%</td>
<td>10%</td>
</tr>
<tr>
<td>4</td>
<td>Individual shares</td>
<td>3.6</td>
<td>6.7</td>
<td>9.4</td>
<td>11.7</td>
<td>13.7</td>
<td>18.0</td>
</tr>
<tr>
<td></td>
<td>$\Delta H$</td>
<td>26.6</td>
<td>90.1</td>
<td>174.9</td>
<td>272.0</td>
<td>375.9</td>
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<td>5</td>
<td>Individual shares</td>
<td>4.4</td>
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<td>11.0</td>
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<td>15.8</td>
<td>20.2</td>
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<td>39.4</td>
<td>128.6</td>
<td>242.0</td>
<td>366.9</td>
<td>496.3</td>
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<tr>
<td>6</td>
<td>Individual shares</td>
<td>5.2</td>
<td>9.2</td>
<td>12.5</td>
<td>15.2</td>
<td>17.5</td>
<td>22.1</td>
</tr>
<tr>
<td></td>
<td>$\Delta H$</td>
<td>54.0</td>
<td>170.0</td>
<td>311.4</td>
<td>461.9</td>
<td>613.8</td>
<td>978.9</td>
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</table>

of the market demand elasticity shows how screening thresholds should interact with market definition, a point we return to in Section 5.

**MNL demand.** In the MNL case, the demand for product $k$ can be written as

$$D^k(p^k; A) = \frac{\exp\left(\frac{b^k-p^k}{\lambda}\right)}{A},$$

where the aggregator $A$ now takes the form

$$A \equiv \sum_{j \in N} \exp\left(\frac{b^j-p^j}{\lambda}\right) + A^0.$$
and $\lambda > 0$ is a price sensitivity parameter.\textsuperscript{33} As in the CES case, consumer surplus is equal to $\log A$.

From the first-order conditions of profit maximization, firm $f$ sets the same absolute markup $\mu_f > 0$ on each of its products,\textsuperscript{34}

$$p^j - c^j = \mu_f \quad \forall j \in f. \tag{15}$$

As total consumption (including the outside good) is equal to one, firm $f$’s market share is naturally measured in volume (rather than value), and given by

$$s_f \equiv \sum_{j \in f} D^j(p^j; A).$$

The markup and market share fitting-in functions are the unique solutions in $\mu_f$ and $s_f$ to the following system of equations:

$$\mu_f = \frac{\lambda}{1 - \frac{T_f}{A} \exp \left( -\frac{\mu_f}{\lambda} \right)}, \tag{16}$$

$$s_f = \frac{T_f}{A} \exp \left( -\frac{\mu_f}{\lambda} \right), \tag{17}$$

where

$$T_f \equiv \sum_{k \in f} \exp \left( \frac{b^k - c^k}{\lambda} \right)$$

is firm $f$’s type.

We are interested in the synergies required for merger $M$ between firms $m$ and $n$ not to harm consumers. First, we state the MNL-analog of Proposition 2:

**Proposition 3.** With MNL demand, for merger $M = \{m, n\}$ to be CS-neutral, the merger-induced type synergy has to satisfy

$$\frac{T_M}{T_m + T_n} = \frac{s_M \exp \left( \frac{1}{1-s_M} \right)}{s_m \exp \left( \frac{1}{1-s_m} \right) + s_n \exp \left( \frac{1}{1-s_n} \right)}, \tag{18}$$

where $s_M \equiv s_m + s_n$ is the market share of the merged firm.

**Proof.** From equations (16) and (17), we obtain

$$\frac{T_f}{A^*} = s_f \exp \left( \frac{1}{1 - s_f} \right). \tag{19}$$

\textsuperscript{33}The own-price elasticity of a product $j$ is $\epsilon^j = (1 - s^j)p^j/\lambda$, while the aggregate elasticity for the inside goods is $\epsilon = s_0p/\lambda$, where $s^0$ is the market share of the outside good and $p$ is the quantity-weighted average price of the inside goods.

\textsuperscript{34}The equilibrium analysis here follows again Nocke and Schutz (2018).
Hence, for merger $M$ to be CS-neutral, the post-merger type $T_M$ has to satisfy

$$\frac{T_M}{A^*} = s_M \exp \left( \frac{1}{1 - s_M} \right).$$

(20)

Combining (19) and (20), yields equation (18).

Proposition 3 shows that with MNL demand, the required synergies again do not depend on the level of concentration among the non-merging firms and are larger for larger mergers. In addition, holding $s_M$ fixed, a more asymmetric merger again requires fewer type synergies:

**Corollary 4.** With MNL demand, a sum-preserving spread of the merger partners’ pre-merger market shares (which decreases the change in the Herfindahl index) reduces the type synergies required to prevent consumer harm.

*Proof.* This follows from the convexity of $s \exp(1/(1-s))$ in $s$, implying that a sum-preserving spread of $s_m$ and $s_n$ increases the denominator on the right-hand side of equation (18).

Assuming that the merger affects only marginal costs, the following corollary relates the size of the required (absolute) marginal cost changes to pre-merger market shares:

**Corollary 5.** Suppose that the set of products offered—and the associated qualities—are not affected by merger $M = \{m, n\}$.

(i) With MNL demand, prices are unaffected by the merger if and only if for each merging firm $k = m, n$ the absolute change in the marginal cost of each of firm $k$’s products $j \in k$, $\Delta c^j$, satisfies

$$\Delta c^j = -\frac{\lambda (s_M - s_k)}{(1 - s_M)(1 - s_k)}.$$

(ii) With MNL demand, if the marginal cost of each product $j \in (m \cup n)$ changes by the same absolute amount, $\Delta c$, consumer surplus remains unchanged if and only if

$$\Delta c = -\lambda \log \left( \frac{s_M \exp \left( \frac{1}{1-s_M} \right)}{s_m \exp \left( \frac{1}{1-s_m} \right) + s_n \exp \left( \frac{1}{1-s_n} \right)} \right).$$

*Proof.* See Appendix B.2.

Assuming no outside good ($A^0 = 0$), Table 3 depicts the maximal individual shares and change in the Herfindahl index for a symmetric-firm merger at various synergy levels. The upper part of the table measures synergies in terms of percentage change in type (as did the upper part of Table 2 for CES demand).

The lower part of the table measures synergies in terms of percentage change in marginal cost (as did the lower part of Table 2 for CES demand). However, recall from Corollary 5
that – under MNL demand – what is pinned down, for a given price sensitivity parameter $\lambda$, are the required absolute rather than relative cost changes. Nonetheless, in the special case in which all firms and products are symmetric, we can derive a condition for the required percentage cost synergy $\phi$ for a given price elasticity of firm-level demand:

$$\phi = \frac{-\Delta c}{c} = \frac{\lambda}{c} \left( \frac{s}{(1-2s)(1-s)} \right) = \left( \frac{1}{\epsilon_f - 1} \right) \left( \frac{s}{1-2s} \right), \quad (21)$$

where $c$ is the common pre-merger marginal cost, $\epsilon_f$ is the common firm-level elasticity of demand, and $s$ is the common firm-level market share. The second equality follows from Corollary 5, and the third from equations (15)-(17), yielding $c = p - \lambda/(1 - s)$, and the fact that $\epsilon_f = (1 - s)p/\lambda$. The individual shares reported in the lower part of the table are the solutions in $s$ (times 100) of equation (21).

**Table 3:** Maximal Level of Individual Shares and $\Delta H$ (*10,000) To Prevent Consumer Harm for Various Levels of Type Synergy (Upper Panel) and Cost Synergy (Lower Panel) with MNL Demand. The lower panel assumes symmetric firms and products.

<table>
<thead>
<tr>
<th>Type Synergy:</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>7.5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual shares</td>
<td>1.0</td>
<td>1.9</td>
<td>2.7</td>
<td>3.5</td>
<td>4.3</td>
<td>6.0</td>
<td>7.5</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>1.9</td>
<td>7.0</td>
<td>14.8</td>
<td>24.8</td>
<td>36.5</td>
<td>71.6</td>
<td>112.4</td>
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<table>
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<tr>
<th>Cost Synergy:</th>
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<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>7.5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_f$</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>1%</td>
<td>2%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>Individual shares</td>
<td>2.8</td>
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<td>15.5</td>
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<tr>
<td>$\Delta H$</td>
<td>16.0</td>
<td>57.4</td>
<td>116.3</td>
<td>187.3</td>
<td>266.3</td>
<td>481.6</td>
<td>703.1</td>
</tr>
<tr>
<td>Individual shares</td>
<td>3.7</td>
<td>6.9</td>
<td>9.7</td>
<td>12.1</td>
<td>14.3</td>
<td>18.8</td>
<td>22.2</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>27.4</td>
<td>95.1</td>
<td>187.3</td>
<td>293.8</td>
<td>408.2</td>
<td>703.1</td>
<td>987.7</td>
</tr>
<tr>
<td>Individual shares</td>
<td>4.5</td>
<td>8.3</td>
<td>11.5</td>
<td>14.3</td>
<td>16.7</td>
<td>21.4</td>
<td>25.0</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>41.3</td>
<td>138.9</td>
<td>266.3</td>
<td>408.1</td>
<td>555.6</td>
<td>918.4</td>
<td>1250.0</td>
</tr>
</tbody>
</table>

Like for the CES model, Table 3 shows that, at common firm-level elasticities, lower cost synergies are required in the MNL model than in the Cournot model. Nonetheless, as in the CES model, even mergers among small firms would require significant synergies for the merger not to harm consumers. For example, a merger between two firms with a 10% pre-merger market share each (raising the Herfindahl index by 200) would require type synergies exceeding 10%, and cost synergies exceeding 3% when the firm-level own-price elasticity is 5.
As in the CES case, if there is an outside good, with share $s^0$, the critical share levels recorded in Table 3 would need to be adjusted by the factor $1/(1 - s^0)$. For the MNL case, the elasticity formulas in footnote 33 imply that $s^0 = (1 - s^f)\epsilon/\epsilon^f$, where $\epsilon$ is the aggregate price elasticity of the inside goods (the “market demand elasticity”). Thus, $s^0 \leq \epsilon/\epsilon^f$. For example, if $\epsilon = 1.5$ and $\epsilon^f = 5$, the critical shares in Table 3 would increase by at most a factor of 1.43.

4 Empirical Analysis of Hypothetical Mergers in Brewing

The theoretical results above suggest that the presence of consumer harm from a horizontal merger may be more strongly related to the change in the Herfindahl than to its post-merger level. However, the models of Section 3 are very special, and our results for them also leave some possibility for the level of the post-merger Herfindahl to be related to the synergies required to prevent consumer harm through its relation to aggregate conditions such as the market elasticity of demand in the Cournot model or the outside good share in the MNL and CES models.

In this section, we take a different approach, by looking empirically at how the synergy required to prevent consumer harm is related to the level and merger-induced change in the Herfindahl index for various hypothetical mergers in the U.S. brewing industry.\textsuperscript{35}

We focus on the brewing industry because markets for beer are local, giving us many hypothetical mergers with varying market shares and market conditions, and because prior work by Miller and Weinberg (2017) has estimated a demand system and marginal costs for the major beer brands. We consider the estimates from Miller and Weinberg’s RCNL-1 and RCNL-3 monthly models, random-coefficient nested logit models that are not covered by our analysis in Section 3.\textsuperscript{36} We use these demand estimates, Miller and Weinberg’s derived region/brand-specific marginal costs, and the values of the exogenous determinants of demand in each region in January 2005 (the first month of the Miller and Weinberg estimation sample) to simulate each possible hypothetical merger among the producers in each of Miller and Weinberg’s 39 local markets.\textsuperscript{37} Given the five firms in their estimation

\textsuperscript{35}Garmon (2017) and Hosken et al. (2018) examine predictors of post-merger price changes in actual hospital and grocery mergers, respectively. However, they have very small samples (28 mergers in the former paper and 14 mergers in the latter paper).

\textsuperscript{36}The difference between these two models is in the product attributes that are given random coefficients. In RCNL-1, price, calories, and a constant receive random coefficients that depend on a consumer’s income. In RCNL-3, import status and package size (the two key determinants of price) receive random coefficients instead of price. In general, for a given package size, import status is the key product characteristic leading the demand estimates to diverge from the identical cross elasticity across inside goods that characterizes a simple nested logit model (see Miller and Weinberg’s Table V for the RCNL-1 model, and Table I.1 in http://www.nathanhmiller.org/Miller%20Weinberg%20(Supplement).pdf). For this reason, we regard the RCNL-3 estimates as being further away from a multinomial logit model.

\textsuperscript{37}Miller and Weinberg (2017) provide evidence that tacit coordination emerged following the 2008 approval
model, this gives 10 possible mergers in each local market, for a total of 390 hypothetical mergers.

For each possible merger and a given specified synergy for the merging firms (which reduces the pre-merger marginal costs of each of the merging firms’ products by the same percentage), we compute the pricing equilibrium and resulting consumer welfare. We do this for various possible synergy levels, and identify the synergy level at which the merger is CS-neutral. As well, we calculate the post-merger Herfindahl index and the change in the Herfindahl for that merger, with the shares for this computation including all firms in the market, not just the five firms in the Miller and Weinberg estimation model. We report results based on volume shares in the main text, and provide results based on revenue shares in Appendix C. (Overall, the results are very similar.) We then examine how these two characteristics of mergers are related to the required synergy across our hypothetical mergers.

Figures 5 and 6 plot the results for the RCNL-1 and RCNL-3 models, respectively. Each small symbol represents a merger and its location shows that merger’s post-merger Herfindahl index and the merger-induced change in the Herfindahl. Green crosses indicate mergers whose required efficiency gain is 0-5%; brown squares indicate those with a required gain between 5% and 10%, blue circles between 10% and 15%, and red diamonds above 15%. The visually striking aspect of the figure is that whether a merger would require less than a 5% efficiency gain to avoid harming consumers is highly related to the change in the Herfindahl, and seems nearly unrelated to the level of the post-merger Herfindahl (and, if anything, holding fixed the change, increases in the level of the Herfindahl appear to require lower efficiency gains to prevent consumer harm). Thus, holding the shares of the merging firms fixed, increases in the level of concentration of the non-merging firms in a market appear to have little effect on the required efficiency gain.

Miller and Weinberg include only the flagship brands of the five firms in their demand model. With the other brands of these firms implicitly included in the outside good, the price elevation arising in our merger simulations is likely less than would be the price elevation were all of these firms’ products included as inside goods.

The fact that the largest values for $\Delta H$ in these mergers generally increase in the level of $\overline{H}$ is expected: For a merger $M = \{m, n\}$ of firms with shares $s_m$ and $s_n$, we have $\overline{H} = \sum_{j \notin M} (s_j)^2 + (s_m + s_n)^2 \geq \sum_{j \notin M} (s_j)^2 + 2\Delta H$, so $\Delta H \leq \overline{H}/2$. 

of the MillerCoors joint venture. Given our focus on unilateral effects, we rely on Miller and Wenberg’s demand and cost estimation results, but impose static Nash price-setting behavior in conducting our merger simulations.
Figure 5: Relationship between the synergy required for a merger to be CS-neutral and the post-merger Herfindahl index and its change, based on the RCNL-1 model and volume shares.
Figure 6: Relationship between the synergy required for a merger to be CS-neutral and the post-merger Herfindahl index and its change, based on the RCNL-3 model and volume shares.
Table 4: Regression of the Required Synergy on Functions of the Herfindahl and the Change in the Herfindahl (Volume-based)

<table>
<thead>
<tr>
<th>Dependent Variable: Synergy Required to Prevent Consumer Harm</th>
<th>RCNL-1 (1)</th>
<th>RCNL-3 (4)</th>
<th>RCNL-1 (2)</th>
<th>RCNL-3 (5)</th>
<th>RCNL-1 (3)</th>
<th>RCNL-3 (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hhi</td>
<td>−.013</td>
<td>.038</td>
<td>−.795</td>
<td>.047</td>
<td>−.352</td>
<td>.292</td>
</tr>
<tr>
<td></td>
<td>(.032)</td>
<td>(.047)</td>
<td>(.202)</td>
<td>(.292)</td>
<td>−2.50</td>
<td>(.218)</td>
</tr>
<tr>
<td></td>
<td>[.03]</td>
<td>[.81]</td>
<td>[−3.93]</td>
<td>[−3.85]</td>
<td>[−2.09]</td>
<td></td>
</tr>
<tr>
<td>delta</td>
<td>2.39</td>
<td>3.14</td>
<td>3.21</td>
<td>4.18</td>
<td>2.68</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>(.062)</td>
<td>(.089)</td>
<td>(.302)</td>
<td>(.044)</td>
<td>(.310)</td>
<td>(.480)</td>
</tr>
<tr>
<td></td>
<td>[38.89]</td>
<td>[35.36]</td>
<td>[10.62]</td>
<td>[9.58]</td>
<td>[8.65]</td>
<td>[6.52]</td>
</tr>
<tr>
<td>hhi × delta</td>
<td>−4.44</td>
<td>−4.44</td>
<td>−4.17</td>
<td>−4.44</td>
<td>−3.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(2.00)</td>
<td>(1.01)</td>
<td>(1.55)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[−3.21]</td>
<td>[−2.22]</td>
<td>[−4.15]</td>
<td>[−2.46]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hhi$^2$</td>
<td>1.79</td>
<td>2.56</td>
<td>.81</td>
<td>1.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.45)</td>
<td>(.65)</td>
<td>(.30)</td>
<td>(.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4.00]</td>
<td>[3.96]</td>
<td>[2.73]</td>
<td>[2.30]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta$^2$</td>
<td>3.77</td>
<td>1.71</td>
<td>9.98</td>
<td>13.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.66)</td>
<td>(2.41)</td>
<td>(2.33)</td>
<td>(3.61)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.27]</td>
<td>[3.70]</td>
<td>[2.48]</td>
<td>[3.70]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>−.002</td>
<td>−.016</td>
<td>.077</td>
<td>.102</td>
<td>.036</td>
<td>.045</td>
</tr>
<tr>
<td></td>
<td>(.008)</td>
<td>(.011)</td>
<td>(.022)</td>
<td>(.032)</td>
<td>(.017)</td>
<td>(.026)</td>
</tr>
<tr>
<td></td>
<td>[−.26]</td>
<td>[−1.47]</td>
<td>[3.47]</td>
<td>[3.20]</td>
<td>[2.17]</td>
<td>[1.75]</td>
</tr>
</tbody>
</table>

Sample | Full | Full | Restricted | Full | Full | Restricted | Full | Full | Restricted |
# Observations | 390 | 390 | 352 | 390 | 390 | 352 | 390 | 390 | 352 |
$R^2$ | .85 | .86 | .82 | .83 | .84 | .77 |

Notes: Dependent variable measured as 0.01 for 1% synergy, hhi is the volume-based post-merger Herfindahl index scaled between 0 and 1, and delta is the merger-induced change in the volume-based Herfindahl index scaled between 0 and 1. Standard errors are in parentheses; $t$-statistics are in square brackets.

Columns (1) and (4) of Table 4 confirm this impression, reporting on the results of a simple linear regression of the synergy required to make a merger CS-neutral on a merger’s post-merger Herfindahl index (referred to as “hhi” in the table), the change in the Herfindahl caused by the merger (referred to as “delta” in the table), and a constant. For both RCNL models, the change in the Herfindahl is strongly significant while the level of the post-merger Herfindahl is insignificant and small in magnitude. For example, the RCNL-3 estimated coefficient on the post-merger Herfindahl implies that a 1000 point increase in the post-merger
Herfindahl causes only a 0.38 percentage point increase in the synergy required for consumers to not be harmed. In contrast, the estimated coefficient on the change in the Herfindahl implies that an extra 100 points for the change leads to a 3.14 percentage point increase in the required synergy. Note also that the $R^2$ of both of these regressions is remarkably high, equalling 0.85 in column (1) and 0.83 in column (4).

Columns (2) and (5) of Table 4 explore this relationship further by expanding the specification to include second-order terms in hhi and delta. Columns (3) and (6) then restrict the sample to the 352 mergers for which the post-merger Herfindahl is less than 4000 and the change in the Herfindahl is less than 1000, which is both where most of the data lies and the region where screening and presumption thresholds are likely most relevant. F-tests for all of these estimations strongly reject both the simple linear model and a model in which all terms involving the post-merger Herfindahl index are dropped.

In all four regressions, a greater increase in the Herfindahl increases the synergy required for consumers not to be harmed. In contrast, while the post-merger Herfindahl does matter in these second-order specifications, its effect is not monotonic and its magnitude is often small. To see this point, Figures 7 and 8 plot contour lines for the estimates in the restricted samples of columns (3) and (6). We plot contour lines for synergies of 1%, 3%, and 5%. Thus, for example, if a merger has a 3% synergy, those mergers lying above the 3% line are CS-decreasing, and those lying below it are CS-increasing.

As can be seen in the two figures, the effect of the post-merger Herfindahl is quite small for Herfindahl levels between 1500 and 2500, where a merger is expected to leave consumers unharmed if the change in the Herfindahl is somewhere in the 150-180 range. For levels of the post-merger Herfindahl both below and above this range, lower changes in the Herfindahl are required for consumers to be unharmed; only at very high levels of concentration do increases in the Herfindahl make a merger much more likely to lead to consumer harm (for a given size of the merger-induced change). Notice, as well, that if a 3% efficiency gain is presumed, any merger that induces an increase of more than 200 in the Herfindahl index is expected to harm consumers, regardless of the level of the post-merger Herfindahl index. The results here suggest a threshold for the change in the Herfindahl somewhere in the middle of those suggested by the theoretical models of Section 3 (compare Tables 1-4).

Figures 9 and 10 compare the consumer welfare effects for these 390 hypothetical mergers of the 1968, 1982 and 2010 Guidelines’ thresholds to both the optimal approval rule (which approves a merger if and only if it increases consumer surplus) and the optimal rule based on only the change in the Herfindahl index. For the latter, for a given presumed efficiency gain (0-5%) we find the cut-off threshold for the change in the HHI that maximizes consumer welfare, when mergers below the threshold are approved and those above it are rejected. For the 1982 and 2010 Guidelines, we assume that mergers in the green zone are approved,

$^{40}$For the RCNL-1 estimates, these cutoffs are 0, 37, 103, 163, 210, 269 for 0%, 1%, 2%, 3%, 4% and 5% presumed efficiency gains, respectively. For the RCNL-3 estimates, these cutoffs are 0, 24, 103, 113, 184, 206 for 0%, 1%, 2%, 3%, 4% and 5% presumed efficiency gains, respectively.
Figure 7: Contour plot showing the combinations of the post-merger Herfindahl (labelled here as “post_hhi_vol”) and the merger-induced change in the Herfindahl (“delta_hhi_vol”) that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 4, column (3).
Figure 8: Contour plot showing the combinations of the post-merger Herfindahl (labelled here as “post_hhi_vol”) and the merger-induced change in the Herfindahl (“delta_hhi_vol”) that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 4, column (6).
Figure 9: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta HHI$ (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and volume shares.
Figure 10: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and volume shares.
that mergers in the red zone are rejected, and that mergers in the yellow zone have the correct decision 75% of the time. For the 1968 Guidelines, we assume mergers are blocked in the cases identified in Figure 1, and otherwise allowed. The horizontal axis measures the uniform percentage price change for the products in our demand system that generates the same change in consumer surplus as the merger policy. Thus, the optimal policy always has a negative equivalent price change, as the possibility of allowing mergers under that policy always weakly increases welfare. The equivalent price change grows more negative as the efficiency gains increase, reflecting increasing benefits for consumers.

The striking aspect of the figures is how close to the welfare gains of the optimal policy is the policy based on only the change in HHI, and how much better that policy is for these hypothetical beer mergers than either the 1982 or 2010 Guidelines’ policies, despite the fact that we assume that under the Guidelines’ policies the correct decision is reached in the yellow zone three quarters of the time. The figures also show that, for these mergers, only when the efficiency gains reach 5% do the Guidelines’ policies generate positive consumer gains, and that the 1982 thresholds are better for consumers than the 2010 ones if efficiency gains are less than 5% (and roughly equal at 5%), and that the 1968 Guidelines are better than the 1982 Guidelines if efficiency gains are 3% or less.

Finally, we examine the effect on consumers of the mergers that fall into each of the 2010 Guidelines’ green, yellow, and red zones (recall Figure 3). Table 5 presents these statistics for the case in which mergers result in a 3% synergy. Several clear points come out. First, a very high share of the mergers in which the post-merger Herfindahl is below 1500, which fall in the Guidelines’ safe harbor, lead to consumer harm: 63% for the RCNL-1 model and 68% in the RCNL-3 model. On the other hand, mergers in the safe harbor zone at higher levels of the post-merger Herfindahl rarely harm consumers. Second, nearly all mergers in the red zone harm consumers. Finally, mergers in the yellow zone – which the 2010 Guidelines’ consider potentially problematic – often lead to consumer harm. In Table 6 we report the same information under the presumption that mergers lead to a 5% synergy. Consistent with Figures 9 and 10, the table shows that with this larger presumed synergy the Guidelines’ thresholds are more successful at sorting good and bad mergers among this set of brewing mergers.

\[^{41}\text{As described in the 1968 Guidelines, we linearly extrapolate between the cases in Figure 1, assuming that the larger firm is the acquirer.}\]

\[^{42}\text{Figures assuming a correct decision in the yellow zone 85% and 95% of the time for the 1982 and 2010 Guidelines are in Appendix E. The } \Delta \text{HHI-based policy continues to outperform the 1982 and the 2010 Guidelines even in the 95% case.}\]

\[^{43}\text{One important caveat is that the characteristics of the hypothetical mergers in our sample may not correspond to the distribution of mergers that would actually be proposed to the agencies. Indeed, even when mergers are profitable (as all are here), which mergers get proposed is the result of both negotiations/bidding among firms in an industry, and the treatment firms expect from the agencies (see, for example, Nocke and Whinston, 2010 and 2013).}\]
Table 5: Share of Hypothetical Brewing Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 Guidelines’ Screening Thresholds (Volume-based)

<table>
<thead>
<tr>
<th>Merger Guidelines’ Screening Zone</th>
<th>RCNL-1</th>
<th>RCNL-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Zone (Safe Harbor)</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>HHI &lt; 1500</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Yellow Zone</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &gt; 100</td>
<td>0.76</td>
<td>0.85</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH ∈ (100, 200)</td>
<td>0.28</td>
<td>0.76</td>
</tr>
<tr>
<td>Red Zone (Anticompetitive Presumption)</td>
<td>0.98</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 6: Share of Hypothetical Brewing Mergers with 5% Efficiency Gain That Harm Consumers Under 2010 Guidelines’ Screening Thresholds (Volume-based)

<table>
<thead>
<tr>
<th>Merger Guidelines’ Screening Zone</th>
<th>RCNL-1</th>
<th>RCNL-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Zone (Safe Harbor)</td>
<td>0.08</td>
<td>0.12</td>
</tr>
<tr>
<td>HHI &lt; 1500</td>
<td>0.24</td>
<td>0.37</td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Yellow Zone</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &gt; 100</td>
<td>0.49</td>
<td>0.60</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH ∈ (100, 200)</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td>Red Zone (Anticompetitive Presumption)</td>
<td>0.84</td>
<td>0.95</td>
</tr>
</tbody>
</table>

5 Discussion

The theoretical and empirical results above suggest that when screening mergers for whether their unilateral price effects will harm consumers, the merger-induced change in the Herfindahl index should play a much more prominent role than the level of the Herfindahl. As well, they indicate that at common firm and market-level elasticity levels, if the typical merger were to have a 3% efficiency gain, consumer harm would often arise even for mergers leading to a 100-200 point increase in the Herfindahl index. In this section, we discuss two issues raised by these results: (i) Are there other factors that might lead screening mergers based on the level of the post-merger Herfindahl index to make sense?, and (ii) Are the screening thresholds in the 2010 Guidelines too lax?
5.1 When Might Screening Based on the Level of the Herfindahl Index Make Sense?

Our analysis has maintained several assumptions: we focused only on harm from unilateral price effects, we considered only pricing responses by rivals, we assumed that efficiency gains are unrelated to market structure, and we required that consumers not be harmed. Relaxing any of these assumptions could, perhaps, open the door for screening based on the Herfindahl index to make sense.

One possibility, of course, is that screening based on the level of post-merger concentration might be justified by concerns over coordinated, rather than unilateral, price effects. For example, in the simplest repeated Bertrand model of price competition coordinated effects arise only once a critical level of concentration is reached. That said, we know of no literature at present that provides general results on how, when coordinated effects are possible, the level of the Herfindahl index is related to the efficiency gains necessary to prevent consumer harm from a merger. Moreover, as we noted earlier, merger investigations have focused largely on unilateral effects in recent years.

A second possibility is that introducing non-price responses by rivals, such as product repositioning or entry, could lead to different conclusions, and perhaps create a role for the Herfindahl index in properly screening mergers. While this possibility exists, we do not have any sense that these considerations would favor reliance on the level of the Herfindahl index.

A third possibility is that the efficiency gains in a typical merger may themselves be related to the level of the Herfindahl index. For example, a merger in an unconcentrated industry may require greater efficiencies to be profitable than one in a concentrated industry. It would be interesting to see to what extent this type of effect might restore a role for the level of the Herfindahl index in screening for unilateral price effects, as well as whether there is empirical support for such an effect.

A fourth possible reason for focusing on the level of the Herfindahl index could be that merger authorities (courts and/or agencies) either have a different objective than preventing consumer harm or face additional constraints. For example, an authority’s objective might instead reflect a desire to prevent significant consumer harm. Indeed, the idea that increases in concentration lead to greater and greater increases in price is one intuitive argument for being concerned with the post-merger level of the Herfindahl index. Note, however, that it assumes that consumers will be harmed by allowed mergers.

In the Cournot model, for example, the magnitude of any merger-induced reduction in consumer surplus depends on the characteristics of the non-merging outsiders. Formally, the derivative of consumer surplus with respect to the merged firm’s post-merger marginal cost,

---

\[ \frac{\partial CS}{\partial MC} \]

\[ \text{Indeed, as we noted in footnote 12, there is some reason to believe that the focus on the level of the Herfindahl index in the 1982 Guidelines may have been partly for this reason.} \]

\[ \text{One piece of evidence that merger efficiencies may be related to market structure and the market-power-enhancing effect of a merger is Schmitt (2017)’s finding that targets in out-of-market hospital mergers experience efficiency gains, while targets in within-market hospital mergers do not. However, it is suggestive more of efficiencies being related to increases in market power (\( \Delta \text{HHI} \)) than to the level of market power.} \]

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evaluated at the level at which the merger would just be CS-neutral, equals

\[
dCS(Q^*) \frac{dQ}{d\sigma_M} = -\frac{Q^*}{|F| - \sigma(Q^*)},
\]

where \(\sigma(Q) \equiv -QP''(Q)/P'(Q) < 1\) is the curvature of inverse demand and \(|F|\) is the pre-merger number of active firms. Hence, at a given pre-merger equilibrium output level \(Q^*\), the reduction in consumer surplus when efficiencies fall short of the level that would leave consumers surplus unchanged is smaller the larger is the number of firms. This fact implies that if an antitrust authority’s goal is to ensure that the post-merger CS-level is at least a fraction \(x\) of the pre-merger level, with \(x\) strictly less than (but close to) one, then the required merger-induced efficiencies are decreasing in the number of firms. The key force driving this effect is that with fewer rival firms, non-merging firms replace less of any reduction in the merging firms’ supply.

Similarly, under price competition with CES/MNL demands, the concentration among outsiders’ market shares—a kin to the Herfindahl index—comes into play, as the following proposition shows:

**Proposition 4.** Assume that the market share of each non-merging firm does not exceed 0.65. Then, with CES or MNL demand, a sum-preserving spread of the market shares of the non-merging firms makes consumer surplus more responsive to the level of merger-induced efficiencies.

**Proof.** See Appendix B.3.

We explored this possibility in our beer data by performing a similar analysis to that in Section 4, but instead focusing on the level of synergy required to prevent a merger from causing more than a 5% reduction in consumer surplus. We found evidence of a positive effect of the level of the post-merger Herfindahl when using the RCNL-1 estimates, but not when using the RCNL-3 estimates.

A different possibility is that antitrust agencies face the need to protect consumers given a limited enforcement budget. In that situation, the agencies would want to focus on the worst mergers for consumers. To explore this avenue, we looked at the relationship in our beer data between the absolute size of a merger’s effect on consumer surplus and the levels of the Herfindahl index, its merger-induced change, and market size for a 3% presumed marginal cost synergy. We found that both the change in the Herfindahl and market size strongly predicted the absolute level of consumer harm from a merger, but there was no significant effect of the level of the post-merger Herfindahl once these other variables were controlled for.\(^{46}\)

\(^{46}\)Another possibility is that a focus on the Herfindahl is appropriate if the authority is concerned about aggregate, rather than consumer, surplus (despite the law’s focus on consumer harm). Farrell and Shapiro (1990) and Nocke and Schutz (2019) discuss some aspects of the relationship between the level of the Herfindahl and the aggregate surplus effect of a merger, but we are unaware of any results about the overall relationship.
In sum, we do not discount the possibility that, in some circumstances, screening mergers in part based their resulting post-merger level of the Herfindahl index may make sense. Yet, at the same time, we view our results as raising the bar for the level of theoretical and empirical support that should back up any such claim.

5.2 Are Current Merger Screening Thresholds Too Lax?

The results above indicate that the current Guidelines’ thresholds, when applied to the kind of ‘natural’ markets that arise in litigated cases, are likely too lax for preventing consumer harm from unilateral price effects unless one is willing to credit the typical merger with a 5% or greater reduction in marginal cost, or believe that product repositioning, entry, or other effects not considered here will be effective at constraining these price effects.47,48

This observation leads naturally to the question of what the evidence is concerning efficiency gains from horizontal mergers, particularly those likely to be near screening thresholds. While casual observation and the agencies’ skepticism about efficiency claims suggest that 5% is rather optimistic for most mergers, there is remarkably little solid empirical evidence on this point. Whinston (2007) summarizes work on the topic as of 15 years ago. Much of the work since then on productivity effects of mergers across a range of industries has examined effects on revenue productivity (TFPR), which cannot distinguish between market power and true efficiency effects.49 A recent exception is Blonigen and Pierce (2016), who use the methods of De Loecker and Warzynski (2012) to identify market power and productivity levels for manufacturing plants in the U.S., and examine the effects of mergers on these measures. Overall, they find evidence for significant effects of mergers on market power but no evidence for an effect on plant-level productivity.50 Moreover, for mergers that have a horizontal aspect (measured as being in the same 2-digit or four-digit SIC industry), they find negative productivity effects.

There have also been a couple of more recent analyses focusing on specific mergers or groups of mergers in specific industries. Ashenfelter et al. (2015) and Miller and Weinberg (2017) document large reductions in marginal cost from the reduced shipping needs resulting from the MillerCoors joint venture.51 Kulick (2017) studies mergers in the ready-mix concrete

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47 The market-level elasticities we focused on in Section 3, and market shares measured in a broad beer market in Section 4, reflected this focus on natural market definitions. As we argued at the end of Section 2, such market definitions appear to be what the agencies need to claim when going to court, and hence what is relevant for assessing the stringency of current Guidelines’ thresholds.

48 Throughout this subsection, we consider whether the current Guidelines are too lax to prevent consumer harm. It is possible that they may be too lax in this sense, but the best the agencies can do (in terms of stringency) with too small a budget.

49 Máksimovic and Phillips (2001), for example, examine TFPR; see Foster et al. (2008) for an examination of the issue.

50 In principle, there could be marginal cost reductions arising from mergers in the absence of plant-level efficiency gains, such as because of better allocation of production across plants or reductions in input prices that had been inflated due to supplier market power.

51 The inference in these two papers is, however, indirect: reductions in shipping distance post-merger are shown to be related to reductions in retail prices.
industry. He finds that acquired plants in mergers of firms with closely located plants experience roughly a 6% productivity gain, but cannot reject that acquirers experience no gain.\footnote{Kulick’s has only 20 acquirer plants in the same local area as the plants being acquired, so the standard error of the acquirer estimate is large. In his “benchmark” results, the point estimate for acquirers is roughly a 2% gain. The horizontal mergers with local overlap in Kulick’s sample generally involve a firm with one plant acquiring multiple rivals over a 5-10 year period. Given the ratio of acquiring plants to acquired plants, the acquiring firm appears to be about four times larger than the acquired plant in the average merger. If so, this 2% point estimate would mean that the average merger has roughly a 3% quantity-weighted efficiency gain.} He also finds that prices rose as a result of these mergers. Schmitt (2017) finds no evidence for post-merger cost reductions in a sample of within-market hospital mergers in 2000-2010.\footnote{In addition, Braguinsky et al. (2015) study the Japanese cotton spinning industry at the turn of the 20th century. The firms in this industry had little market power, and the industry saw a wave of acquisitions over 30 years. Braguinsky et al. (2015) show that these acquisitions led to productivity improvements, on average, of almost 13% in the acquired plants. The productivity improvement for acquisitions by serial acquirers was, on average, almost 16%.}

Based on the results we report above, our sense of the agencies’ experience with mergers, and our reading of the current (meager) evidence in the literature, we conclude that existing thresholds are likely to be too lax, a conclusion that also finds support in the more extensive merger-retrospective evidence on price effects of horizontal mergers (e.g., Ashenfelter and Hosken, 2010, Kwoka, 2015). We believe this is especially true with regard to safe harbors, since mergers that fall into these categories are simply allowed, while those that are above these thresholds are investigated in greater detail and may still be (and often are) allowed. That said, there is a clear need for much better evidence on the efficiency effects to be expected from mergers near the screening thresholds to better support such a conclusion.

6 Conclusion

In this paper we have explored the use of concentration measures to screen horizontal mergers for unilateral price effects. Looking both theoretically and empirically, our results suggest that such screens should likely focus much more on the merger-induced change in the Herfindahl index than on its post-merger level. As such, they suggest screens closer in form to the 1968 Guidelines than to the current ones. In terms of stringency, our results indicate that the thresholds in the current 2010 Guidelines are likely too lax, given the markets the agencies typically allege in litigation, unless one expects efficiency gains of 5% or greater from the typical merger, or other factors such as entry and product repositioning to significantly constrain the exercise of market power post-merger.\footnote{One alternative to more stringent thresholds would be to allege narrower markets. For example, if the agencies actually alleged the narrowest markets satisfying the hypothetical monopolist test, Appendix D shows that many mergers would likely be considered 2-to-1 mergers. We are skeptical, however, that this strategy would be successful in litigation.}

We see several useful directions for further work to refine concentration screens for horizontal mergers. First, further empirical analysis along the lines of that in Section 4
other markets with different estimated demand and costs would be very useful. Second, more evidence on the synergies arising in horizontal mergers, especially conditional on market structure, would be extremely valuable. Third, work identifying thresholds for screening mergers for possible consumer harm due to coordinated effects would complement our analysis. Finally, continuing work on merger retrospectives is important, especially aimed at learning both the extent to which entry, repositioning, or other factors on average ameliorate unilateral anti-competitive effects, and the extent to which coordinated effects arise that exacerbate them.\footnote{The prospects for such merger retrospectives would be significantly enhanced if merging firms were required to provide post-merger data to the agencies.}

At the same time, of course, concentration screens are just one piece of the merger evaluation puzzle. It is easy to come up with examples where concentration measures alone would lead to incorrect conclusions about whether a proposed merger would harm consumers. Nonetheless, they can prove useful when more detailed information on margins, diversion ratios and cost synergies are unknown or of uncertain reliability, provided that they are well-designed in both form and stringency. In this paper, we have aimed to shed light on how to do that.

References


Appendix

A Merger Enforcement Statistics

Table 7: FTC horizontal merger enforcement statistics, 1996-2011: Percent of mergers enforced among those receiving a second request, as a function of the post-merger level of the Herfindahl index and the merger-induced change in the Herfindahl index. The total number of mergers receiving a second request are shown in brackets. [Source: U.S. Federal Trade Commission (2013).]

<table>
<thead>
<tr>
<th>Change in HHI:</th>
<th>0-99</th>
<th>100-199</th>
<th>200-299</th>
<th>300-499</th>
<th>500-799</th>
<th>800-1199</th>
<th>1200-2499</th>
<th>2500+</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-1799</td>
<td>0%</td>
<td>35.4%</td>
<td>48.7%</td>
<td>60.7%</td>
<td>30.0%</td>
<td>0.0%</td>
<td>—</td>
<td>—</td>
<td>40.0%</td>
</tr>
<tr>
<td>1800-1999</td>
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<td>45.5%</td>
<td>75.0%</td>
<td>70.6%</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
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<td>46.7%</td>
<td>56.8%</td>
<td>72.7%</td>
<td>50.0%</td>
<td>—</td>
<td>—</td>
<td>58.1%</td>
</tr>
<tr>
<td>2400-2999</td>
<td>33.3%</td>
<td>66.7%</td>
<td>54.5%</td>
<td>75.0%</td>
<td>75.9%</td>
<td>72.2%</td>
<td>—</td>
<td>—</td>
<td>71.7%</td>
</tr>
<tr>
<td>3000-3999</td>
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<td>60.0%</td>
<td>71.4%</td>
<td>64.3%</td>
<td>64.1%</td>
<td>77.2%</td>
<td>73.6%</td>
<td>—</td>
<td>71.5%</td>
</tr>
<tr>
<td>4000-4999</td>
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<td>50.0%</td>
<td>83.3%</td>
<td>71.4%</td>
<td>81.8%</td>
<td>95.8%</td>
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<td>100%</td>
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<td>100%</td>
<td>91.3%</td>
<td>87.9%</td>
<td>90.4%</td>
<td>89.5%</td>
</tr>
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<td>—</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>96.3%</td>
<td>99.2%</td>
<td>99.0%</td>
</tr>
<tr>
<td>Total</td>
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<td>44.7%</td>
<td>54.2%</td>
<td>66.9%</td>
<td>72.5%</td>
<td>78.6%</td>
<td>88.0%</td>
<td>97.7%</td>
<td>77.6%</td>
</tr>
</tbody>
</table>

B Proofs

B.1 Proof of Corollary 3

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$c^k = p^k \left[1 - \frac{1}{\sigma(1 - s_m) + s_m}\right],$$
where we have used equations (7) and (10). For the merged firm to charge the same prices for all of its products (implying that its post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product $k$ has to satisfy

$$ \bar{c}^k = p^k \left[ 1 - \frac{1}{\sigma(1 - s_M) + s_M} \right]. $$

Combining, we obtain:

$$ \phi^k = \frac{\bar{c}^k - c^k}{c^k} = -\frac{s_M - s_m}{(1 - s_m)[\sigma(1 - s_M) + s_M]}.$$

To see part (ii), note that:

$$ \bar{T}_M / (T_m + T_n) = \frac{\sum_{k \in M} b^k ((1 - \phi)c^k)^{1-\sigma}}{\sum_{k \in m} b^k (c^k)^{1-\sigma} + \sum_{k \in n} b^k (c^k)^{1-\sigma}} $$

$$ = \frac{(1 - \phi)^{1-\sigma} \sum_{k \in M} b^k (c^k)^{1-\sigma}}{\sum_{k \in m} b^k (c^k)^{1-\sigma} + \sum_{k \in n} b^k (c^k)^{1-\sigma}} $$

$$ = (1 - \phi)^{1-\sigma}. $$

The assertion then follows from applying Proposition 2.

**B.2 Proof of Corollary 5**

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$ c^k = p^k - \frac{\lambda}{1 - s_m}, $$

where we have used equations (15) and (16) For the merger to leave all prices unchanged (implying that the post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product $k$ has to satisfy

$$ \bar{c}^k = p^k - \frac{\lambda}{1 - s_M}. $$

Combining, we obtain:

$$ \Delta c^k = \bar{c}^k - c^k = \frac{\lambda}{1 - s_m} - \frac{\lambda}{1 - s_M} = \frac{\lambda(s_M - s_m)}{(1 - s_M)(1 - s_m)}. $$

To see part (ii), note that:

$$ \frac{\bar{T}_M}{T_m + T_n} = \frac{\sum_{k \in M} \exp \left( \frac{b^k - c^k - \Delta c}{\lambda} \right)}{\sum_{k \in m} \exp \left( \frac{b^k - c^k}{\lambda} \right) + \sum_{k \in n} \exp \left( \frac{b^k - c^k}{\lambda} \right)}. $$
\[ \text{exp} \left( \frac{-\Delta c}{\lambda} \right) \sum_{k \in M} \text{exp} \left( \frac{b_k - c_k}{\lambda} \right) = \frac{\text{exp} \left( \frac{-\Delta c}{\lambda} \right) \sum_{k \in m} \text{exp} \left( \frac{b_k - c_k}{\lambda} \right) + \sum_{k \in n} \text{exp} \left( \frac{b_k - c_k}{\lambda} \right) + \sum_{f \notin M} \text{exp} \left( \frac{b_f - c_f}{\lambda} \right) }{\sum_{k \in m} \text{exp} \left( \frac{b_k - c_k}{\lambda} \right) + \sum_{k \in n} \text{exp} \left( \frac{b_k - c_k}{\lambda} \right) + \sum_{f \notin M} \text{exp} \left( \frac{b_f - c_f}{\lambda} \right) + A^0 A^*} \]

The assertion then follows from applying Proposition 3.

### B.3 Proof of Proposition 4

If the post-merger type \( T_M \) falls short by a small fraction of the level that would restore consumer surplus after the merger, the shortfall in consumer surplus is given by

\[
- dCS(A^*) \frac{dA}{dT_M} T_M = - \frac{T_M A^* S'(T_M)}{A^*} + \sum_{f \notin M} T_f A^* S'(T_f) = - \frac{S^{-1}(s_M) S'(S^{-1}(s_M)) + \sum_{f \notin M} S^{-1}(s_f) S'(S^{-1}(s_f))}{S^{-1}(s_M) S'(S^{-1}(s_M)) + \sum_{f \notin M} S^{-1}(s_f) S'(S^{-1}(s_f))}
\]

where the first equality follows from applying the implicit function theorem to the adding-up condition

\[
S' \left( \frac{T_M}{A^*} \right) + \sum_{f \notin M} S' \left( \frac{T_f}{A^*} \right) + \frac{A^0}{A^*} = 1.
\]

As the number of outsiders is finite, it is straightforward to see that a sum-preserving spread of the outsiders’ market shares can be decomposed into a finite number of steps where at each step there is a sum-preserving spread of market shares involving only two outsiders. We now prove that at any such step the denominator on the r.h.s. of equation (22) decreases, from which the result follows.

Let \( t_f \equiv T_f/A^* \) and suppose that \( t_f > t_g \). We need to show that an increase in \( t_f \) and a decrease in \( t_g \) such that \( S(t_f) + S(t_g) \) remains unchanged induces a reduction in \( t_f S'(t_f) + t_g S'(t_g) \). We have:

\[
\frac{d}{dt_f} \left[ t_f S'(t_f) + t_g S'(t_g) \right] \bigg|_{S(t_f)+S(t_g)=\text{const.}} = S'(t_f) \left[ t_f S''(t_f) S'(t_f) - \frac{t_g S''(t_g)}{S'(t_g)} \right].
\]

As \( S'(\cdot) > 0 \), we thus only need to show that the elasticity of \( S' \) is decreasing, i.e.,

\[
\frac{d}{dt} \frac{t S''(t)}{S'(t)} < 0.
\]
From the proof of Proposition 9 in Nocke and Schutz (2019), we have:

\[
S'(t) = \frac{1}{t} \frac{S(t)(1 - S(t))(1 - \alpha S(t))}{1 - S(t) + \alpha S(t)^2},
\]
\[
S''(t) = -\frac{\alpha (2 - S(t))S(t)^2(1 - S(t))(1 - \alpha S(t))}{[1 - S(t) + \alpha S(t)^2]^3},
\]

where \( \alpha = 1 \) if demand is of the MNL form and \( \alpha = (\sigma - 1/\sigma) < 1 \) if it is of the CES form.

It follows that

\[
\frac{tS''(t)}{S'(t)} = -\frac{\alpha(2 - S(t))S(t)}{[1 - S(t) + \alpha S(t)^2]^2}.
\]

We thus have

\[
\frac{d}{dt} \frac{tS''(t)}{S'(t)} < 0
\]

if and only if

\[
[(2 - S(t))S'(t) - S(t)S'(t)][1 - S(t) + \alpha S(t)^2] > 2(2 - S(t))S(t)[S'(t) + 2\alpha S(t)S'(t)],
\]

i.e.,

\[
1 + \alpha S(t)^3 > 3\alpha S(t)^2.
\]

It can easily be verified that this inequality holds, for any \( \alpha \in (0, 1] \) if \( S(t) \leq 0.65 \).

C Empirical Results for Brewing Mergers using Revenue Shares

Here we present the tables and figures for the empirical analysis of Section 4 when markets shares and the Herfindahl index are revenue-based rather than volume-based.
Table 8: Regression of the Required Synergy on Functions of the Herfindahl and the Change in the Herfindahl (Revenue-based)

<table>
<thead>
<tr>
<th></th>
<th>RCNL-1</th>
<th></th>
<th>RCNL-3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
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<td>.020</td>
<td>−.813</td>
<td>−.635</td>
<td>.082</td>
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<tr>
<td></td>
<td>(.032)</td>
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<td>(.045)</td>
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<td>[.62]</td>
<td>[−4.46]</td>
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<tr>
<td>delta</td>
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<td>2.72</td>
<td>2.41</td>
<td>3.01</td>
</tr>
<tr>
<td></td>
<td>(.060)</td>
<td>(.270)</td>
<td>(.325)</td>
<td>(.087)</td>
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<td>[10.09]</td>
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<td>−3.58</td>
<td></td>
<td>−2.34</td>
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<tr>
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<td>(1.17)</td>
<td>(1.08)</td>
<td></td>
<td>(1.71)</td>
</tr>
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<td>[−2.44]</td>
<td>[−3.30]</td>
<td></td>
<td>[−1.37]</td>
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<td>hhi²</td>
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<td>2.38</td>
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<td>(.36)</td>
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<td>(.54)</td>
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<td>[.30]</td>
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<td>(.020)</td>
<td>(.011)</td>
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<td>[3.96]</td>
<td>[3.44]</td>
<td>[−2.96]</td>
</tr>
</tbody>
</table>

Sample | Full | Full | Restricted | Full | Full | Restricted

# Observations | 390 | 390 | 343 | 390 | 390 | 343

| R²    | .85 | .86 | .79 | .83 | .84 | .74 |

Notes: Dependent variable measured as 0.01 for 1% synergy, hhi is the revenue-based post-merger Herfindahl index scaled between 0 and 1, and delta is the merger-induced change in the revenue-based Herfindahl index scaled between 0 and 1. Standard errors are in parentheses; t-statistics are in square brackets.
Figure 11: Contour plot showing the combinations of the post-merger Herfindahl (labelled here as “post_hhi_val”) and the merger-induced change in the Herfindahl (“delta_hhi_val”) that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 8, column (5).
**Figure 12**: Contour plot showing the combinations of the post-merger Herfindahl (labelled here as “post_hhi_val”) and the merger-induced change in the Herfindahl (“delta_hhi_val”) that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 8, column (6).
Figure 13: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta \text{HHI}$ (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and value shares.
Figure 14: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on ΔHHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and value shares.
Table 9: Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 Guidelines’ Screening Thresholds (Revenue-based)

<table>
<thead>
<tr>
<th>Merger Guidelines’ Screening Zone</th>
<th>RCNL-1</th>
<th>RCNL-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Green Zone (Safe Harbor)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI &lt; 1500</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &lt; 100</td>
<td>0.50</td>
<td>0.61</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Yellow Zone</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &gt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH ∈ (100, 200)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Red Zone (Anticompetitive Presumption)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.76</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 10: Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 Guidelines’ Screening Thresholds (Revenue-based)

<table>
<thead>
<tr>
<th>Merger Guidelines’ Screening Zone</th>
<th>RCNL-1</th>
<th>RCNL-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Green Zone (Safe Harbor)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI &lt; 1500</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &lt; 100</td>
<td>0.17</td>
<td>0.28</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH &lt; 100</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Yellow Zone</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI ∈ (1500, 2500) and ΔH &gt; 100</td>
<td>0.48</td>
<td>0.58</td>
</tr>
<tr>
<td>HHI &gt; 2500 and ΔH ∈ (100, 200)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Red Zone (Anticompetitive Presumption)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.76</td>
<td>0.89</td>
</tr>
</tbody>
</table>

D Implications of the Hypothetical Monopolist Test

In this appendix, we study the implications of the hypothetical monopolist test (HMT) for market definition—and the relationship to the conditions under which a merger does not hurt consumers. We do so in two theoretical models: in the homogeneous-goods Cournot model and in the CES model of differentiated-goods price competition.
D.1 Implications of the HMT in the Cournot Model

Suppose we have a symmetric Cournot model with \(|F|\) firms and a constant elasticity demand function \(P(\sum_i q_i)\) with elasticity \(\varepsilon\) and cost per unit \(c\). The individual firm first-order condition is

\[
(P(Q) - c) + P'(Q)q_i = 0 \tag{23}
\]

Imposing symmetry (\(q_i = q\) for all \(i\)) and rearranging we get the equilibrium condition:

\[
\frac{p - c}{p} = \frac{1}{|F|\varepsilon} \tag{24}
\]

so

\[
p = \left(\frac{|F|\varepsilon}{|F|\varepsilon - 1}\right) c \tag{25}
\]

or

\[
p - c = \left(\frac{1}{|F|\varepsilon - 1}\right) c. \tag{26}
\]

**Upward pricing pressure.** The upward pricing pressure (really downward output pressure) created by a merger of two symmetric firms is \(P'(Q)q\). So a merger will not reduce output if the merger-induced improvement in cost per unit \((-\Delta c)\) is less than \(P'(Q)q\), which can be rewritten, using (23), as\(^{56}\)

\[
|\Delta c| > (p - c)
\]

or, substituting from (26) and rearranging,

\[
\frac{|\Delta c|}{c} > \left(\frac{1}{|F|\varepsilon - 1}\right)
\]

For an efficiency improvement that reduces marginal cost to \((1 - e)c\), this would be satisfied for \((|F|, \varepsilon)\) such that

\[
|F|\varepsilon > 1 + e \tag{27}
\]

**The HMT.** The HMT restricts what can qualify as a relevant market, and hence implicitly the \((|F|, \varepsilon)\) elasticities we might be dealing with. Roughly, a market qualifies as relevant/valid if a merger of all firms would result in a price increase of at least 5\%.\(^ {57}\) This can be stated

\(^{56}\)A merger of two symmetric firms results in the derivative of the firm’s profit taking value \((P(Q) - (c + \Delta c)) + P'(Q)2q_i\) if quantities don’t change. Using (23), this derivative is negative (resulting in the merged firm having an incentive to reduce output) if \(-\Delta c + P'(Q)q_i < 0\).

\(^{57}\)More accurately, it says that a hypothetical monopolist would find a 5% price increase profitable. So the condition we use above is somewhat more restrictive when monopoly profit is a concave function (the monopolist could optimally increase price less than 5% but still find a 5% increase profitable).
as:

\[(1.05) \left( \frac{\vert \mathcal{F} \vert \varepsilon}{\vert \mathcal{F} \vert \varepsilon - 1} \right) c < \left( \frac{\varepsilon}{\varepsilon - 1} \right) c\]

or equivalently,

\[1.05 \vert \mathcal{F} \vert (\varepsilon - 1) < (\vert \mathcal{F} \vert \varepsilon - 1)\]

Alternatively, given \(\varepsilon\) the HMT is passed if

\[\vert \mathcal{F} \vert > \frac{1}{1.05 - 0.05\varepsilon}\]  \hspace{1cm} (28)

**Putting these together.** Figure 15 shows these two relationships together for the case of \(e = 0.03\). CS-decreasing mergers are those that lie below the hatched (red) curve \(\vert \mathcal{F} \vert \varepsilon = 34.33\), while the homogeneous-goods market passes the HMT to the left of the solid (blue) curve (where equation (28) is satisfied). As is evident in the figure, for elasticities below 11, the HMT is always satisfied and so for the elasticities in typical “natural” markets asserted in merger cases, the HMT would always be passed.

Figure 15 also allows us to assess, when demand takes a constant elasticity form, what insisting on the narrowest HMT-supported market would imply. To do this we ask what the smallest subset of firms is that would pass the HMT (this exercise can be thought of as considering a case in which the firms’ products are very close to homogeneous, and selecting a subset of firms is choosing a smallest set of firms that are close to the merging firms).

Note that when demand takes a constant elasticity form, when we conduct the HMT holding fixed the outputs of the “out of market” firms, the residual demand elasticity facing the “in market” firms is still \(\varepsilon\). Hence, Figure 15 can be used to find, given \(\varepsilon\), the smallest number of firms satisfying the HMT. As Figure 15 makes clear, if \(\varepsilon\) is less than 11, this would always consist of a two-firm market (i.e., the narrowest relevant market would consist of the merging firms).

Similar conclusions hold if one considers instead efficiency gains of 5%.

### D.2 Implications of the HMT in the CES Model of Price Competition

Consider an industry consisting of \(\vert \mathcal{F} \vert\) symmetric firms, each of type \(T\). Denoting by \(\mu \equiv (p^k - c^k)/p^k \in (0, 1)\) the (common) percentage markup on any product \(k\), and \(\sigma > 1\) the elasticity of substitution, we have for any firm \(f \in \mathcal{F}\):

\[
\sum_{k \in f} b^k (p^k)^{1-\sigma} = \sum_{k \in f} b^k (c^k)^{1-\sigma} \left( \frac{c^k}{p^k} \right)^{\sigma-1} = T_f (1 - \mu_f)^{\sigma-1} = T (1 - \mu)^{\sigma-1},
\]
Figure 15: The symmetric-firm Cournot model with constant price elasticity of demand, with the elasticity depicted on the horizontal axis and the number of firms on the vertical axis. The HMT is satisfied is to the left of the solid (blue) curve; assuming an efficiency of 3 percent ($e = 0.03$), a two-firm merger is CS-decreasing below the hatched (red) curve.
where the last equality follows by symmetry. The equilibrium aggregator \( A \) can therefore be written as
\[
A = A^0 + |F|T(1 - \mu)^{\sigma-1},
\]
where \( A^0 > 0 \) is the value of the outside good.

In the following, we normalize \( A^0 \equiv 1 \), and let \( \varepsilon = \sigma - (\sigma - 1)(1 - s^0) \in (1, \sigma) \) denote the (pre-merger) market-wide price elasticity, and \( s^0 = A^0/A \) the (pre-merger) market share (in value) of the outside option. Hence, the (pre-merger) equilibrium value of the aggregator is given by
\[
A^* = \frac{\sigma - 1}{\varepsilon - 1}.
\]
As \( A^* = 1 + |F|T(1 - \mu^*)^{\sigma-1} \), we obtain
\[
T = \frac{\sigma - \varepsilon}{(\varepsilon - 1)|F|(1 - \mu^*)^{\sigma-1}}.
\]
The equilibrium markup \( \mu^* \) is uniquely determined by
\[
\sigma \mu^* \left( 1 - \frac{\sigma - 1}{\sigma} \cdot \frac{\varepsilon}{\varepsilon - 1} \right) = 1.
\]
Rewriting,
\[
\mu^* = \frac{|F|}{(|F| - 1)\sigma + \varepsilon},
\]
implying
\[
T = \frac{\sigma - \varepsilon}{(\varepsilon - 1)|F|} \left( \frac{(|F| - 1)\sigma + \varepsilon}{(|F| - 1)\sigma + \varepsilon - |F|} \right)^{\sigma-1}.
\]
The (pre-merger) joint equilibrium profit is given by
\[
\Pi^* = \mu^*(1 - s^0) = \frac{(\sigma - \varepsilon)|F|}{(\sigma - 1)[(|F| - 1)\sigma + \varepsilon]}.
\] (29)

**The HMT.** Consider now an increase in all prices by 5 percent. The resulting markup is
\[
\hat{\mu} = \frac{0.05 + \mu^*}{1.05}.
\]
The joint profit is then given by
\[
\hat{\Pi} = \hat{\mu} \cdot \frac{|F|T(1 - \hat{\mu})^{\sigma-1}}{1 + |F|T(1 - \hat{\mu})^{\sigma-1}} = \frac{0.05 + \mu^*}{1.05} \cdot \frac{|F|T(1 - \mu^*)^{\sigma-1}}{1.05^{\sigma-1} + |F|T(1 - \mu^*)^{\sigma-1}}.
\]
Rewriting,

\[ \hat{\Pi} = 0.05 + \frac{|F|}{(\frac{|F| - 1}{\sigma})} \cdot \frac{(\sigma - \varepsilon)}{(1 - \varepsilon)} \cdot \frac{1.05^{\sigma - 1}}{1.05^{\sigma - 1} + (\frac{\sigma - \varepsilon}{(\varepsilon - 1)})} \]

\[ = 0.05 + \frac{|F|}{(\frac{|F| - 1}{\sigma})} \cdot \frac{\sigma - \varepsilon}{(\varepsilon - 1)1.05^{\sigma - 1} + (\sigma - \varepsilon)} \]

\[ = \frac{|F|}{(\sigma - 1)} \cdot \frac{0.05[(|F| - 1)\sigma + \varepsilon]}{(\varepsilon - 1)1.05^{\sigma} + 1.05(\sigma - \varepsilon)}. \]

The price increase is (weakly) profitable if and only if \( \Pi^* \leq \hat{\Pi} \), i.e.,

\[ \frac{|F|}{(\sigma - 1)} \leq \frac{|F|}{(\varepsilon - 1)1.05^{\sigma} + 1.05(\sigma - \varepsilon)}. \]

i.e.,

\[ |F|(\varepsilon - 1)1.05^{\sigma} + 1.05N(\sigma - \varepsilon) \leq (\sigma - 1) \frac{|F| + 0.05[(|F| - 1)\sigma + \varepsilon]}{(\varepsilon - 1)1.05^{\sigma} + 1.05(\sigma - \varepsilon)}. \] (30)

**Upward pricing pressure.** Now, let us consider the conditions under which a merger between two firms does not hurt consumers. Let \( T \) and \( \mu \) denote the post-merger type and markup (of the merged firm), and let \( t \equiv T/(2T) \). For the merger to be CS-neutral requires that

\[ 2T(1 - \mu^*)^{\sigma - 1} = T(1 - \mu)^{\sigma - 1}, \]

i.e.,

\[ \bar{\mu} = 1 - t^{-\frac{1}{\sigma - 1}}(1 - \mu^*). \]

It also requires that

\[ \sigma \bar{\mu} \left( 1 - \frac{\sigma - 1}{\sigma} \frac{1 - \bar{T}(1 - \mu)^{\sigma - 1}}{A^*} \right) = 1. \]

Combining, the merger is CS-neutral if and only if

\[ \sigma \left[ 1 - t^{-\frac{1}{\sigma - 1}} \frac{(|F| - 1)\sigma + \varepsilon - |F|}{(|F| - 1)\sigma + \varepsilon} \right] \left[ 1 - \left( \frac{\sigma - \varepsilon}{\sigma} \right) \frac{2}{|F|} \right] = 1, \]

or

\[ t^{-\frac{1}{\sigma - 1}} = \left( \frac{|F| - 1)\sigma + \varepsilon - |F|}{(|F| - 1)\sigma + \varepsilon} \right) \left( \frac{|F| - 2)\sigma + 2\varepsilon}{(|F| - 2)\sigma + 2\varepsilon - |F|} \right). \]

Let us now replace type efficiencies by cost efficiencies: suppose that the merged firm produces the same set of products as the merger partners did jointly before the merger but that the post-merger marginal cost of any product \( k \) satisfies \( \bar{c}^k = (1 - e)c^k \), where \( c^k \) is the pre-merger marginal cost. The post-merger type \( \bar{T} \) then satisfies

\[ \bar{T} = 2(1 - e)^{1-\sigma}T, \]
Figure 16: The symmetric-firm CES model with $\sigma = 5$. The market-wide price elasticity of demand is depicted on the horizontal axis and the number of firms on the vertical axis. The HMT is satisfied to the left of the solid (blue) curve; assuming an efficiency of 3 percent ($e = 0.03$), a two-firm merger is CS-decreasing below the hatched (red) curve.

so that $t = (1 - e)^{1-\sigma}$. Hence, the merger is CS-neutral if and only if

$$
(1 - e) = \left( \frac{(|F| - 1)\sigma + \epsilon}{(|F| - 1)\sigma + \epsilon - |F|} \right) \left( \frac{(|F| - 2)\sigma + 2\epsilon - |F|}{(|F| - 2)\sigma + 2\epsilon} \right); 
$$

(31)

it is CS-decreasing if the l.h.s. is larger than the r.h.s., and CS-increasing if the l.h.s. is smaller than the r.h.s.

Putting these together. For a given level of $\sigma$ and $e$, we can now plot (30) and (31) in $(|F|, \epsilon)$-space. Figure 16 shows the case of $\sigma = 5$ and $e = 0.03$. Here the entire market passes the HMT provided that the market-wide elasticity is less than 4 (and in any case in which the merger would reduce consumer surplus), and the narrowest HMT-supported market consists of just the merging firms in all of these cases. As in the Cournot case, similar conclusions hold if one considers instead efficiency gains of 5%. 
E Performance of Alternative Merger Approval Policies

Figure 17: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 Guidelines) is correct with 85% probability.
Figure 18: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 Guidelines) is correct with 95% probability.
Figure 19: Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 Guidelines’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta \text{HHI}$ (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 Guidelines) is correct with 85% probability.
**Figure 20:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*’ thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on ΔHHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 *Guidelines*) is correct with 95% probability.