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A THEORY OF CLEARANCE SALES*

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Clearance sales are widely used by firms as an intertemporal selling policy, in particular in markets where firms face demand uncertainty and need to choose capacity in advance. Clearance sales consist in charging a high price initially but then lowering the price in the sales period. High-valuation consumers purchase the good at the high initial price so as to avoid rationing at the low price, while low-valuation consumers wait for the price to drop. We develop a simple model of intertemporal monopoly pricing under demand uncertainty, and show that clearance sales may be the optimal intertemporal selling policy.

In this article, we derive the optimal selling policy of a monopolist who faces uncertainty about the level of demand. In contrast to much of the literature, we show that uniform pricing is not necessarily optimal. The optimal selling policy may instead involve *clearance sales*: the monopolist commits to supply only a limited quantity, and then charge a high price in the first period (palatable to high-valuation consumers) and a low price (affordable to low-valuation consumers) in the last period. The premium associated with the rent offered to consumers that buy at the high price and the probability of being rationed at the low price induce all high-valuation consumers to separate themselves and purchase the good in the first period. In contrast, low-valuation consumers purchase the good in the last period but are rationed with an optimally determined probability.

Clearance sales are commonly used by retailers selling season goods. Durable goods such as winter or summer clothes and seasonal outdoor products (such as skis and camping equipment) are typically liquidated before the season ends. Since producers are limited in their ability to increase production at short notice, sellers have to decide on stocks before the beginning of the season, thus being subject to uncertainty about which items will prove more popular and which less.[1] Unsold items are then marked down in the middle of the season when summer or winter sales typically start. Clearly, consumers anticipate that such a price cut will occur but they are aware of the risk that the particular good they want to purchase may no longer be available by then. Some consumers therefore prefer to buy the good at the regular price before the sales start. Indeed, as a shopping guide puts it:

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[1] For clothing in the 1980s, Pashigian (1988) reports a mean lead time of about 35 weeks for US orders from the Far East, and of more than 14 weeks for domestic orders. More recently, flexible manufacturing and advances in the use of IT have reduced this lead time. This has allowed vertically integrated clothing companies – Zara being the prime example – to replenish stocks rapidly. Interestingly (and consistent with our theory), Zara uses clearance sales only rarely.

> 'There are several things you can do to make shopping for clothes faster, easier and definitely more pleasant. [...] By purchasing at the beginning/middle of the season, you'll have more selections and also be able to purchase coordinates easily. If you [...] wait for end of season sales, you'll be limited [...] in your selection and you may not be able to find coordinates.' (AFE Cosmetics & Skincare Shopping Tips, http://www.cosmetics.com/ward2.htm, checked on May 5, 2005)

Hence, if a firm uses clearances sales, its customers face the trade-off between buying the good at a high price with certainty and waiting for a lower price but then facing the risk of being rationed.

The selling policies of many opera houses, theatres, and concert venues also involve some form of clearance sales (or 'priority pricing'): advance ticket sales at regular prices are complemented by lower-priced 'community rush tickets', 'day seats', or standby tickets. Similarly, holiday tour operators typically offer both regular and last-minute deals. Here, consumers waiting for last-minute holiday packages may not obtain the date, destination, or hotel of choice. In both types of examples, the right to consume a particular product at a given point in time is sold in advance and firms choose intertemporal pricing policies.[2] Firms face demand uncertainty and, well in advance of the date of the performance, they are no longer flexible to adjust capacity to demand.[3] For instance, singers and actors have long-term contractual obligations which do not allow them to give additional performances at short notice. Similarly, tour operators book a certain number of plane seats and hotel rooms well ahead of the beginning of the season, and renegotiation is costly.

Clearance sales have the following properties:

(*i*) the firm charges first a high price and later a lower price (as a discount or mark-down on the regular price), and
(*ii*) in the period of clearance sales, consumers are rationed with positive probability.

The two key features of the markets in which we observe clearance sales are:

(1) the firm faces demand uncertainty, and
(2) the firm has to choose a maximum quantity (capacity) in advance.

As we will show, both features are necessary for clearance sales to be optimal in our model.

We develop a simple model of monopoly pricing and capacity choice under demand uncertainty. There are two types of consumers (with high and low valuations, respectively), two demand states (a good and a bad state), and two periods in which the

[2] A less obvious example of priority pricing concerns season tickets for sporting events (such as baseball or soccer) or cultural events (such as concerts or operas); see Ferguson (1994). While living in London the first author of this article often bought a season ticket for the famous BBC Promenade Concert Series at the Royal Albert Hall, London, knowing that he would attend only a small fraction of the more than seventy concerts. Although the season ticket was therefore more expensive than purchasing 'day seats' for those concerts he would actually attend, his rationale for buying the more expensive season ticket was to make sure that he could attend some of the more popular concerts.

[3] To the extent that firms are flexible to adjust capacity before the season starts, they may have an incentive to offer 'advance-purchase' or 'early-booker' discounts so as to induce some consumers to commit early.

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monopolist may sell the good. Before the state of demand is realised, the monopolist has to choose the overall capacity (quantity ceiling) and a price path.[4] Then, consumers (all of whom want to buy only one unit of the good) learn their own willingness to pay, update their beliefs about the underlying state of the world, and decide when to buy the good. Consumers rationally anticipate the behaviour of other consumers, and thus the endogenous probabilities of rationing in each period.[5]

A uniform pricing policy consists in selling all units at the same price without rationing consumers. Under demand certainty, this would indeed be the monopolist's optimal selling policy. The key result of our article is that, under demand uncertainty, the monopolist may prefer a selling policy involving clearance sales: the monopolist charges a high first-period price and a low second-period price and chooses a capacity level that is less than the demand by high and low types in the good demand state. The low second-period price is chosen so as to extract all of the rents from the low-valuation consumers, while the high first-period price is chosen so as to make high valuation consumers just willing to purchase the good in the first period (rather than wait and be rationed with positive probability at the low second-period price). The clearance sales policy thus allows the monopolist to discriminate effectively between high and low types, and dominates uniform pricing if and only if the monopolist's optimal state-contingent price is high in the good demand state and low in the bad demand state. As we will discuss, the logic of clearance sales applies very generally – and thus extends well beyond our specific model – as long as the monopolist faces demand uncertainty and needs to commit in advance to capacity.

In our model, a second and stronger point can be made. A clearance sales policy can be compared not only to uniform pricing but also to other intertemporal pricing policies. In particular, a firm may want to set a low first-period price and a higher second-period price. Since all consumers prefer to purchase the good at the low initial price, the monopolist needs to ration demand at that price if she wants to make positive sales at the high second-period price and thus be able to discriminate between consumer types. We refer to such a selling policy as an *introductory offer* (or 'advance-purchase discount') policy, which consists in selling a limited quantity at a low price initially, and then raising price. Those consumers with high valuations who were rationed at the lower price may find it optimal to buy the good later at the higher price. While such a policy may perform better than uniform pricing, we show that introductory offers are never optimal in our model: the monopolist's optimal selling policy involves either uniform pricing or clearance sales.

To gain a better understanding of the logic underlying clearance sales, it is useful to consider the following three simple examples. Suppose there are two states of the world, a good demand state and a bad demand state, which occur with probabilities $\Pr(G)$ and $\Pr(B)$, respectively. Consumers have unit demand and may have either a high or a low valuation for the good. High types have a valuation of 1, and low types a valuation of 1/3, independently of the demand state. The monopolist produces at zero cost and may set prices and capacities for two periods. In the case of

[4] Our results would be unaffected if pre-commitment to a future price were infeasible.

[5] If more than one 'consumer equilibrium' exists for a given path of announced prices, we confine attention to the equilibrium that maximises the monopolist's profit.

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excess demand in one period, consumers are rationed randomly. There is no discounting.

Example 1. In the good demand state, there is a mass 2 of high valuation consumers but no low types. In the bad demand state, there are no high types, and a mass 1 of low types. A consumer who learns that he has a high valuation can thus infer that the demand state must be good. The optimal selling policy is a clearance sale with a total capacity of (slightly less than) 2, a first-period price of 1 and a second-period price of 1/3. All high types will then purchase the good in the first period (anticipating that they would be rationed with probability one if they were to wait for clearance sales to occur in the second period), while all low types will purchase the good in the second period. This yields an expected profit of (almost) $\Pr(G) \times 2 + \Pr(B) \times 1/3$. It is straightforward to verify that this clearance sales policy dominates uniform pricing: a uniform price of 1 would yield a profit of only $\Pr(G) \times 2$, while a uniform price of 1/3 would yield an expected profit of only $\Pr(G) \times 2/3 + \Pr(B) \times 1/3$. The clearance sales policy also performs better than the best introductory offer policy, which involves a first-period capacity of 1, a first-period price of 1/3, and a second-period price of 1. This yields an expected profit of only $\Pr(G) \times 4/3 + \Pr(B) \times 1/3$ since, in the good demand state, half of the high types are able to purchase the good at the low first-period price, while the remaining high types are rationed and purchase the good at the high second-period price. In this example, the monopolist can extract all of the surplus by using clearance sales. Such perfect price discrimination is, however, often infeasible.

Example 2. Suppose parameters are as in the first example, but suppose there is an additional mass 1/2 of low-type consumers in each demand state. The optimal selling policy is as in the first example, yielding an expected profit of $\Pr(G) \times 2 + \Pr(B) \times 1/2$. While some consumers with positive valuation do not purchase the good in the good demand state, this clearance sales policy effectively implements the optimal state-contingent pricing policy which involves a price of 1 in the good demand state, and a price of 1/3 in the bad demand state. Hence, as in the first example, the monopolist could not increase her profit further by learning the demand state in advance (before setting prices and capacity).

Example 3. Suppose parameters are as in the first example but suppose there is an additional mass 2 of low-type consumers in each demand state. By choosing the same clearance sales policy as in the previous examples, the monopolist would obtain an expected profit of $\Pr(G) \times 2 + \Pr(B) \times 2/3$. Alternatively, she may choose a clearance sales policy with a total capacity of 3, a first-period price of 2/3 and a second period price of 1/3. Here, the first-period price is less than the high type's valuation of 1 since by postponing the purchasing decision until the second period, a high type is not rationed with probability 1 but with probability 1/2. Expected profit is then $\Pr(G) \times (2 \times 2/3 + 1 \times 1/3) + \Pr(B) \times 3 \times 1/3 = \Pr(G) \times 5/3 + \Pr(B) \times 1$. Hence, this clearance sales policy performs better than the first if and only if the bad demand state is sufficiently likely.

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The optimal clearance sales policy dominates uniform pricing: a uniform price of 1 yields an expected profit of $\Pr(G) \times 2$, while a uniform price of $1/3$ results in an expected profit of $\Pr(G) \times 4/3 + \Pr(B) \times 1$. Clearance sales also perform better than introductory offers. To see this, consider the best introductory offer policy, which consists in setting a first-period capacity of 3, a first-period price of $1/3$, and a second-period price of 1. This policy results in an expected profit of $\Pr(G) \times 3/2 + \Pr(B) \times 1$. While this introductory offer policy may dominate uniform pricing, it is always dominated by a clearance sales policy (namely the one with an overall capacity of 3).

The profit comparison between the different selling policies seems in general rather complex as these policies differ not only in the expected prices the two consumer types have to pay but also in the quantities sold in the different demand states. Using insights from mechanism design theory, however, it can be reduced to a comparison of the implied probabilities of rationing high and low type consumers in the two demand states.

In all three examples, the optimal state-contingent price is equal to the valuation of the high type in the good demand state, and equal to the valuation of the low type in the bad demand state. In the optimal mechanism, high types should always obtain the good with probability 1, while low types should obtain the good with probability 1 in the bad demand state and with probability 0 in the good demand state. In the first two examples, the probabilities of rationing implied by the optimal clearance sales policy are the same as those implied by the optimal mechanism. Uniform pricing and introductory offer policies perform worse since the former implies that the low type's probability of rationing (either 0 under a low uniform price, or 1 under a high uniform price) is independent of the demand state, while the latter policy implies that low types obtain the good with positive probability even in the good demand state.

Matters are more complicated in the third example as the optimal clearance sales policy is not revenue equivalent to the optimal mechanism: while high types obtain the good with probability 1 in each demand state, low types are rationed with probability $1/3$ in the bad demand state if the monopolist commits to a capacity of 2, or – if the monopolist chooses a capacity of 3 instead – they obtain the good with probability $1/2$ even in the good demand state. Nevertheless, clearance sales perform better than other selling policies: the best introductory offer policy implies that the low types obtain the good with probability $3/4$ in the good demand state (and is thus dominated by the clearance sales policy with capacity 3); the low uniform price performs even worse since it implies that the low types obtain the good with probability 1 even in the good demand state; finally, the high uniform price implies that the low types are rationed with probability 1 even in the bad demand state (and is thus dominated by the clearance sales policy with capacity 1).

In the main body of the article, we show that the intuition from these examples holds more generally: the optimal selling policy involves clearance sales if and only if the profit-maximising state-contingent price is high in the good demand state and low in the bad demand state. Otherwise, uniform pricing is optimal.

The article is organised as follows. In the following Section, we discuss the related literature. In Section 2, we present our model. In Section 3, we analyse the monopolist's optimal selling policy. First, we shortly consider the case of demand certainty and show that the optimal selling policy involves uniform pricing. Then, we turn to the

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main concern of this article, monopoly pricing under demand uncertainty, and show that the optimal selling policy involves either uniform pricing or clearance sales. In Section 4, we discuss our key assumptions and model extensions, and argue that clearance sales are likely to emerge in a number of more general settings. We conclude in Section 5.

1. Related Literature

The key result of our article is that a clearance sales policy – and hence a decreasing price path – may be optimal. By contrast, a general theme in the existing literature on intertemporal pricing is that uniform pricing is the profit-maximising selling policy. In a seminal paper on intertemporal pricing under demand certainty, Stokey (1979) has shown that a firm that can commit to a price path optimally sets a uniform price, provided all consumers share the same rate of time preference.[6] For a decreasing price path to obtain, high-valuation consumers must be more 'impatient' than low-valuation consumers.

Another strand of the literature on intertemporal pricing under demand certainty considers capacity constraints – and thus the possibility of rationing – as an additional tool of the monopolist's selling policy. Wilson (1988) analyses the problem of a monopolist who wants to sell q units of a good, each of which may carry a different price tag. When consumers arrive randomly, lower-priced units are sold first. Only when lower-priced units are no longer available, will (high-valuation) consumers purchase a higher-priced unit. If the monopolist chooses to sell at different prices, this selling policy can be interpreted as an introductory offer policy. Wilson shows that an introductory offer policy may be more profitable than uniform pricing (namely if and only if there exists a neighbourhood around q where the single-price revenue function is non-concave in quantity). However, if the monopolist can choose the quantity q she wants to sell, and the marginal cost of production is non-increasing, then uniform pricing is always optimal. Ferguson (1994) introduces the possibility of clearance sales into (the intertemporal version of) Wilson's model, and shows that whenever uniform pricing is not optimal, the best clearance sales policy is revenue equivalent to the best introductory offer policy, for any given quantity q. Hence, the model is unable to explain why a monopolist may strictly prefer clearance sales over introductory offers. Moreover, the results by Wilson (1988) and Ferguson (1994) indicate that non-uniform pricing should be observed only if the single-price revenue function is non-concave in quantity *and* there are decreasing returns to scale in production. In the absence of demand uncertainty, the prevalence of clearance sales is therefore hard to explain.

Harris and Raviv (1981) consider a problem similar to Wilson's but assume that there is demand uncertainty. They allow for more general selling policies, including a 'priority pricing' scheme (akin to clearance sales) where the monopolist announces a price schedule and buyers who announce a willingness to pay a higher price receive priority when the fixed capacity is allocated. Harris and Raviv show that priority pricing is an optimal policy for a monopolist who produces at constant marginal costs but faces a fixed and binding capacity constraint. However, if the monopolist can costlessly choose

[6] Our result on the optimality of clearance sales is robust to the introduction of discounting, see Section 4.

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capacity (and thus serve any demand at constant marginal cost), then uniform pricing dominates other pricing schemes (see also Riley and Zeckhauser, 1983). Harris and Raviv's result relies on a special kind of demand uncertainty: there are a finite number of (large) buyers with i.i.d. valuations. As the number of buyers increases, demand uncertainty vanishes in the limit, and the demand structure becomes a special case of that in Wilson (1988).

Courty (2003) considers a two-period model of ticket pricing, where consumers learn their valuations only after the first period. He shows that the monopolist optimally decides not to ration consumers, and – depending on the level of production costs – all sales are made either in the first period or in the second period. Hence, there is no intertemporal price discrimination.

The above-mentioned papers generate either uniform pricing or else rely on sunk costs (or decreasing returns to scale) to generate non-uniform pricing. The same is true for the literature on price dispersion in competitive markets (Prescott, 1975; Eden, 1990; Dana, 1998).[7] Closely related to our article, Dana (2001) introduces aggregate demand uncertainty into Wilson's (1988) model. He shows that even when the monopolist can produce any quantity at constant marginal cost, she may optimally attach a low price tag to a certain number of items and a high price tag to all other items. That is, introductory offers may dominate uniform pricing. Since lower-priced items are always sold first, however, Dana's model does not allow for clearance sales where some items are marked down after a certain period.

There are two classes of models that generate a decreasing price path. First, Lazear (1986) and Courty and Li (1999) analyse models of monopoly pricing under demand uncertainty and obtain a decreasing price path. In their models, however, consumers are myopic – they do not understand the intertemporal trade-off between buying early and late – and therefore purchase the good whenever the current price is less than their valuation. To the extent that real-world consumers are aware that the firm will mark down all unsold items at a later point in time, these models do not explain clearance sales.

Second, a decreasing price path obtains as an equilibrium outcome in the literature on durable goods pricing by a monopolist who is unable to commit to future prices (Bulow, 1982; Stokey, 1979).[8] However, consumers are never rationed, and so this literature, while generating a decreasing price path, cannot explain clearance sales. Moreover, in the durable goods problem, the monopolist would gain if she could commit to a uniform price. It is worth noting that while in the durable goods literature commitment (to future prices) leads to uniform pricing, in our setup commitment (to total capacity) leads to a decreasing price path rather than uniform pricing. Hence, a decreasing price path is not necessarily an indication of a Coasian commitment

[7] Dana (1999*b*) extends the Prescott model of price dispersion to monopoly and imperfect competition. Gale and Holmes (1993) consider a capacity-constrained monopolist selling two substitute products, a 'peak' and an 'off-peak' flight. They show that introductory offers (or advance-purchase discounts) are optimal. See also Dana (1999*a*) and Gale and Holmes (1992).

[8] In such a setting the possibility of capacity commitment would reduce the price commitment problem of the firm (Kahn, 1986). Furthermore, as Denicolo and Garella (1999) have shown, introductory offers are a useful strategy if the monopolist lacks commitment power beyond the first period but can ration demand in the first period as in Wilson (1988).

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problem; rather, it may be the optimal selling policy of a monopolist who is able to commit to capacity in a world with rational, forward-looking consumers.[9]

2. The Model

We consider a monopolist who sells a homogeneous product over two periods.[10] As in the example of concert tickets and the like, consumption takes place (simultaneously) after period 2. Hence, the time of purchase does not directly affect consumers' utility, and so there is no discounting. More generally, consumers view one unit of the good in period 1 as a perfect substitute for one unit of the good in period 2. That is, we may also think of consumers consuming the good immediately after purchase, provided their discount rate is zero. (As we will discuss in Section 4, our main insights continue to hold with discounting.)

There is a mass M of potential consumers with unit demand. Demand is random and can be in either one of two states $\sigma \in \{G,B\}$: a good demand state, G, and a bad demand state, B, which occur with probabilities $\Pr(G) = \rho$ and $\Pr(B) = 1 - \rho$, respectively. In each demand state, each consumer gets a random draw of his willingness to pay. Consumers can have a high valuation or a low valuation for the good. Moreover, some consumers may not value the good at all; we call such consumers 'null types'. High types are denoted by H, low types by L, null types by $\emptyset$, and the generic consumer type by $\theta \in \{H, L, \emptyset\}$. In demand state σ, the probability that a consumer's type is θ is given by $m(\theta|\sigma)/M$. Here, $m(\theta|\sigma)$ denotes the mass of consumers of type θ in state σ. We assume that the distribution of consumer valuations in the good state first-order stochastically dominates that in the bad state. Specifically, there are (weakly) more high types in the good than in the bad state,

$$m(H|G) \geq m(H|B),$$

and the total mass of consumers with positive valuation is at least as large in the good as in the bad state:

$$m(H|G) + m(L|G) \geq m(H|B) + m(L|B).$$

Also, we require that $m(L|\sigma) > 0$ for $\sigma \in \{G, B\}$.

Conditional on buying one unit of the product at price p, a consumer of type θ has (indirect) utility $v(\theta) - p$, where $v(\theta)$ is the consumer's willingness to pay. High types have a higher willingness to pay than low types, $v(H) > v(L)$. The valuation of a null type is equal to zero, $v(\emptyset) = 0$.[11] We normalise the willingness to pay of the high type (in both demand states) to 1, and so the willingness to pay of the low type satisfies $v(L) \in (0,1)$.

[9] There are also a number of papers on cyclical pricing in a durable goods environment in which in each period new consumers arrive. For instance, Conlisk *et al.* (1984) consider a model with high and low valuation consumers where consumers stay in the market until they purchase the good. In equilibrium, the monopolist periodically sets a lower price to clear out low-valuation consumers. See also Sobel (1984, 1991) and Board (2005).

[10] As we will discuss in Section 4, our analysis would remain unchanged if the monopolist could sell in more than two periods.

[11] Clearly, null types will never purchase the good. We introduce the construct of a null type for technical reasons so as to be able to use Bayesian updating; see footnote 14.

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Before making his purchasing decision, each consumer observes a signal s about the underlying aggregate demand state. In this article, we focus on the information structure, where each consumer only observes his own valuation, i.e., $s = v(\theta)$, and updates his beliefs about the demand state σ using Bayes' rule. The alternative information structure, where each consumer directly observes the true state of demand, i.e., $s = \sigma \in \{G,B\}$, is analysed in our discussion paper, Nocke and Peitz (2004); see Section 4 for a brief discussion.

The monopolist can produce any amount of the homogeneous good at constant marginal cost c. Without loss of generality, we set $c = 0$. The monopolist can sell the product over two periods, $t = 1,2$. Before the demand state is realised,

(*i*) she sets prices p_1 and p_2 for periods 1 and 2, respectively;
(*ii*) she chooses a capacity k (which is the maximum quantity the monopolist may be able to sell over the two periods); and
(*iii*) to allow for introductory offers, she is able to commit not to sell more than a certain fraction of this capacity (namely, $k_1 \leq k$ units) in $t = 1$.

Motivated by the real-world cases discussed in the introduction, we thus assume that prices and capacities cannot be conditioned on the realisation of the demand state.[12] For simplicity, we assume that there are no capacity costs. (As we discuss in Section 4, our main insights hold for any positive and constant unit cost of capacity.)

Depending on the intertemporal profile of prices, we can distinguish between three different types of selling policies:

Uniform pricing. The monopolist sets prices such that, with probability 1, all items are sold at the same price. In particular, setting the same price in both periods, $p_1 = p_2$, is a uniform pricing policy.

Introductory offers. The monopolist sets a lower price in the first period, $p_1 < p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Clearance sales. The monopolist sets a lower price in the second period, $p_1 > p_2$, and some units are sold in each period with positive probability (that is, in at least one demand state).

Since consumers clearly prefer to purchase the good at the lowest possible price, an introductory offer policy must have the property that first-period capacity k_1 is binding in at least one demand state (otherwise all units would be sold at the low price in the first period). Similarly, a clearance sales policy must have the property that total capacity k is binding with positive probability (otherwise, all consumers would always prefer to buy the good at the low price in the second period).

Since the monopolist can commit to capacities, consumers may be rationed in period 1, period 2, or both periods. Following Dana (2001), we assume that rationing is

[12] There are a number of other papers on price discrimination, including Dana (2001), where firms have to commit to prices before demand uncertainty is resolved; see Stole (2001) for a survey.

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proportional (or *random*). Under the proportional rationing rule (Beckmann, 1965; Davidson and Deneckere, 1986), each consumer who is willing to purchase the good has the same probability of obtaining the good. That is, if a mass m of consumers demand the good, but only a quantity $k < m$ is available, then each consumer – independently of his type – is served with probability k/m. Note that this rationing rule is consistent with a queuing model, where consumers arrive in random order, and consumers who arrive first are served first.

Observing the monopolist's policy (p_1, p_2, k_1, k) and their (private) signals about the demand state σ, consumers make their purchasing decisions. For any (p_1, p_2, k_1, k) and distribution of signals, consumers thus play an anonymous game with discrete actions. A consumer equilibrium is a (Bayesian) Nash equilibrium of this (sub-)game. For $p_1 \leq p_2$, consumers have a (weakly) dominant strategy: 'demand the good in the first period if and only if your willingness to pay is equal to or higher than p_1; if you are rationed in the first period, demand the good in the second period, provided your valuation is at least p_2'. Moreover, any consumer equilibrium is revenue equivalent for the monopolist. In contrast, if the monopolist chooses a clearance sales policy (and so $p_1 > p_2$), a consumer may not have a dominant strategy. The only reason why consumers may be willing to buy the good at the higher price in the first period is that they expect to be rationed with a higher probability at the lower price in the second period. However, if consumers expect that more consumers postpone their purchase until $t = 2$, they expect a lower probability of rationing in the second period (as the monopolist will sell all unsold units in the second period), and hence buying in the second period becomes more attractive. This may give rise to the existence of multiple consumer equilibria with different revenues. The best consumer equilibrium from the monopolist's point of view (and the worst from the consumers' point of view) is the one that maximises sales at the high price (in $t = 1$).

The monopolist optimally chooses her policy (p_1, p_2, k_1, k) assuming that, in each subgame, consumers' purchasing decisions form a (Bayesian) Nash equilibrium. To obtain a unique solution, we select, for each (clearance sales) policy of the monopolist, the best consumer equilibrium from the monopolist's point of view.

3. The Optimal Selling Policy

3.1. *Pricing Under Demand Certainty*

To understand the role of demand uncertainty for non-uniform pricing, we first consider the case of demand certainty. Suppose there are no demand shocks, i.e., $m(\theta|\sigma)$ is independent of the state of demand σ. To simplify notation, we write the mass of consumer type θ as $m(\theta)$.

An immediate observation is that we can think of each consumer as choosing between four strategies:

(*i*) 'don't buy';
(*ii*) 'buy in period 1 at price p_1; if rationed, don't buy in period 2',
(*iii*) 'buy in period 1 at price p_1; if rationed, buy in period 2 at price p_2', and
(*iv*) 'don't buy in period 1, but buy in period 2 at price p_2'.

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For any announced selling policy, the consumer's choice problem is therefore essentially static. Any selling policy (p_1, p_2, k_1, k) considered in this article, including uniform pricing, introductory offers, and clearance sales, induces a probability $R(\theta)$ at which a consumer of type θ obtains the good in one of the two periods and an (expected) price $P(\theta)$ conditional on obtaining the good.[13]

It will prove helpful to consider the monopolist's problem from a mechanism design perspective. Since a consumer cannot be forced to make a payment unless he receives the good, we restrict attention to direct-revelation mechanisms that consist of probabilities of receiving the good, $R(H)$ and $R(L)$, and prices conditional on obtaining the good, $P(H)$ and $P(L)$. We will show that, under demand certainty, the solution to the mechanism design problem can be implemented by at least one of the three selling policies. That is, any optimal selling policy implements the solution to the mechanism design problem.

Formally, the monopolist's design problem can be written as

$$\max_{R(H),R(L),P(H),P(L)} m(H)R(H)P(H) + m(L)R(L)P(L),$$

subject to the incentive (IC_θ) and individual rationality (IR_θ) constraints for each type θ. The individual rationality constraint (IR_θ) reflects the fact that, for each selling policy, a consumer can choose to not buy the good and thus not make any payment.

Since the standard single-crossing property is satisfied, it is well known that the solution to such a problem satisfies the (IC_H) and (IR_L) constraints with equality, while the (IC_L) and (IR_H) constraints are nonbinding. Using the binding (IC_H) and (IR_L) constraints, we can rewrite the monopolist's design problem purely in terms of the two probabilities $R(H)$ and $R(L)$:

$$\max_{R(H),R(L)} m(H)R(H) + \{[m(H) + m(L)]v(L) - m(H)\}R(L).$$

The solution to this problem is $R(H) = 1$, and

$$R(L) = \begin{cases} 1 & \text{if } v(L) > m(H)/[m(H) + m(L)] \\ 0 & \text{otherwise.} \end{cases}$$

If the valuation of low types is sufficiently small, $[m(H) + m(L)]v(L) \leq m(H)$, the solution to the mechanism design problem can be implemented by the uniform price $p = 1$, and so only the high-valuation consumers obtain the good. Otherwise, if $[m(H) + m(L)]v(L) \geq m(H)$, the uniform price $p = v(L)$ implements the optimum: all consumers with positive valuation obtain the good with probability 1. We summarise our result in the following proposition.

[13] To see this, let r_t denote the probability at which consumers are rationed in period t. If the consumer chooses 'don't buy', then $R(\theta) = 0$; if he chooses 'buy in period 1 at price p_1; if rationed, don't buy in period 2', then $R(\theta) = 1 - r_1$ and $P(\theta) = p_1$; if he chooses 'buy in period 1 at price p_1; if rationed, buy in period 2 at price p_2', then $R(\theta) = 1 - r_1 r_2$ and $P(\theta) = [(1 - r_1)p_1 + r_1(1 - r_2)p_2]/(1 - r_1 r_2)$; and if he chooses 'don't buy in period 1, but buy in period 2 at price p_2', then $R(\theta) = 1 - r_2$ and $P(\theta) = p_2$.

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Proposition 1. *Under demand certainty, the optimal selling policy is uniform pricing.*

3.2. *Pricing under Demand Uncertainty*

Proposition 1 shows that uniform pricing is always optimal in our model when demand is certain. We now turn to the main concern of this article, the monopolist's optimal selling policy when demand is uncertain.

Since the distribution of consumer types varies with the state of demand, a consumer's best reply to the purchasing strategies of other consumers may depend on his beliefs about the state of demand. Learning his own type, a Bayesian consumer uses this information to update his beliefs about the underlying demand state. However, a consumer's private signal s is typically not perfectly revealing.[14] A consumer who learns that he has a high valuation, will (using Bayes' rule) compute the probability of the state of demand being good as

$$Q(G|H) = \frac{\rho m(H|G)}{\rho m(H|G) + (1-\rho)m(H|B)}.$$

From Proposition 1 we know that if the monopolist could condition her selling policy on the state of demand σ (which, of course, she cannot in our model), she would optimally set a uniform price $p(\sigma)$. The optimal state-contingent (uniform) prices are as follows. In the good demand state, $p(G) = 1$ if the valuation of low types is sufficiently small,

$$m(H|G) > [m(H|G) + m(L|G)]v(L), \tag{1}$$

and $p(G) = v(L)$ if the inequality is reversed. In the bad demand state, $p(B) = v(L)$ if

$$m(H|B) < [m(H|B) + m(L|B)]v(L), \tag{2}$$

and $p(B) = 1$ if the inequality is reversed. In our analysis of the optimal selling policy, these optimal state-contingent prices will provide a useful benchmark.

3.2.1. *A mechanism design perspective*

As in the case of demand certainty, it will prove helpful to consider the related problem of a mechanism designer. We restrict attention to mechanisms which consist, for each consumer type θ and demand state σ, of a probability $R(\theta|\sigma)$ at which the consumer obtains the good in one of the two periods and a price $P(\theta|\sigma)$ conditional on obtaining the good. (Formally, the probabilities and prices do not directly depend on the state of demand but rather on the consumer's report of his own type and the reports of all other consumers. However, if almost all consumers report their types truthfully, the

[14] To use Bayes' rule in our setting, the size of the population of consumers must be independent of the realisation of the state of the world. Since the total mass of high and low type consumers may be larger in the good demand state than in the bad demand state, $m(H|G) + m(L|G) \geq m(H|B) + m(L|B)$, we introduce the construct of a 'null type'. Recall that a null type has a valuation of zero, and is thus not willing to buy at any (positive) price. The mass of null types in demand state σ is the difference between the total mass of consumers and the mass of high and low type consumers, $m(\emptyset|\sigma) = M - [m(H|\sigma) + m(L|\sigma)] \geq 0$. That is, while the total mass of high, low, and null types is independent of the demand state, the shares of the different types are state-dependent.

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monopolist knows the demand state for sure. Abusing notation, we therefore write the probabilities and prices directly as a function of the consumer's own reported type and the state of demand.) In addition to the (interim) incentive constraints (IC_θ) for the high and low types, we require that the mechanism has to satisfy the *ex post* individual rationality constraints ($XIR_{\theta,\sigma}$). To see that the *interim* incentive constraints are relevant, note that a consumer does not know the demand state when making his purchasing decision. To see that the *ex post* individual rationality constraints are relevant, note that, in each demand state, the price a consumer has to pay cannot be larger than his willingness to pay, $P(\theta|\sigma) \leq v(\theta)$. (These *ex post* individual rationality constraints appear reasonable whenever consumers cannot be forced to purchase a good.) Clearly, any selling policy (p_1, p_2, k_1, k) considered in this article, induces probabilities $R(\theta|\sigma)$ and expected prices $P(\theta|\sigma)$ and satisfies the *ex post* individual rationality constraints, and can thus be represented by an element of this class of mechanisms. Below, we will show that the solution to this mechanism design problem is revenue equivalent to the optimal state-contingent prices.

Formally, the monopolist's design problem can be written as

$$\max_{R(\theta|\sigma),P(\theta|\sigma)} \rho[m(H|G)R(H|G)P(H|G) + m(L|G)R(L|G)P(L|G)] \\ + (1-\rho)[m(H|B)R(H|B)P(H|B) + m(L|B)R(L|B)P(L|B)]$$

subject to

(*i*) the (interim) incentive constraints (IC_θ) for each type θ,

$$Q(G|\theta)\{R(\theta|G)[v(\theta) - P(\theta|G)] - R(\overline{\theta}|G)[v(\theta) - P(\overline{\theta}|G)]\} \\ + [1 - Q(G|\theta)]\{R(\theta|B)[v(\theta) - P(\theta|B)] - R(\overline{\theta}|B)[v(\theta) - P(\overline{\theta}|B)]\} \geq 0,$$

where $\overline{\theta} \in \{H,L\}$, $\overline{\theta} \neq \theta$, and

(*ii*) the *ex post* individual rationality constraints ($XIR_{\theta,\sigma}$) for each type θ and demand state σ, $P(\theta|\sigma) \leq v(\theta)$.

It is straightforward to show that the solution to this problem satisfies the high valuation consumer's incentive constraint (IC_H) with equality. Also, the low type's (*ex post*) individual rationality constraints ($XIR_{L,\sigma}$) are binding for both demand states whenever $R(L|\sigma) > 0$.[15] As will become clear later, any selling policy (clearance sales, introductory offers and uniform pricing) that is optimal within its own class (i.e., within the class of clearance sales, introductory offers, or uniform pricing policies) also satisfies the same constraints with equality.[16] Hence, any two such selling policies are revenue equivalent if and only if they induce the same probabilities of serving consumers.

Replacing $P(L|G) = P(L|B) = v(L)$ in (IC_H), we obtain that

[15] If $R(L|\sigma) = 0$, the ($XIR_{L,\sigma}$) constraint becomes irrelevant and $P(L|\sigma)$ cancels out in the monopolist's expected revenue.

[16] The constraint $P(\theta|\sigma) \leq v(\theta)$ holds with equality whenever $R(\theta|\sigma) > 0$. Hence, if the monopolist charges a uniform price $p = 1$, she does not sell to the low types, i.e., $R(L|\sigma) = 0$, and so the constraint becomes irrelevant.

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$$P(H|G) = 1 - \frac{R(L|G)}{R(H|G)}[1 - v(L)] + \frac{1 - Q(G|H)}{Q(G|H)}\left\{\frac{R(H|B)}{R(H|G)}[1 - P(H|B)] + \frac{R(L|B)}{R(H|G)}[1 - v(L)]\right\}.$$

This allows us to write the monopolist's expected revenue purely in terms of probabilities:[17]

$$\begin{aligned}&\rho m(H|G)R(H|G) + \rho\{[m(H|G) + m(L|G)]v(L) - m(H|G)\}R(L|G)\\ &+(1-\rho)m(H|B)R(H|B) + (1-\rho)\{[m(H|B) + m(L|B)]v(L) - m(H|B)\}R(L|B).\end{aligned} \tag{3}$$

Maximising this expression, we obtain that the monopolist should serve high type consumers with probability 1 and low type consumers with probability 0 whenever the optimal state-contingent price is $p(\sigma) = 1$. Similarly, she should serve all consumers with probability 1 whenever the optimal state-contingent price is $p(\sigma) = v(L)$. That is,

$$\begin{aligned} R(H|\sigma) &= 1 \quad \text{for} \quad \sigma = G, B,\\ R(L|G) &= \begin{cases} 0 & \text{if (1) holds} \\ 1 & \text{otherwise,} \end{cases}\\ R(L|B) &= \begin{cases} 1 & \text{if (2) holds} \\ 0 & \text{otherwise.} \end{cases}\end{aligned} \tag{4}$$

When comparing the revenues of our different selling policies, we can thus focus on comparing the implied probabilities of serving the different consumers in the two demand states and analyse how 'close' these are to the probabilities in the optimal benchmark.

We now turn to the main concern of this article, the analysis of the optimal selling policy (p_1, p_2, k_1, k) under demand uncertainty. We first follow Dana (2001) in restricting attention to selling policies with non-decreasing price paths, and compare introductory offers with uniform pricing. Then, we also allow for decreasing price paths and compare clearance sales policies with introductory offers and uniform pricing.

3.2.2. *Uniform pricing*

Let us first consider uniform pricing, where the monopolist sets the same price p in both periods. Under uniform pricing, the monopolist has no incentive to ration consumers, and will thus set capacities $k = k_1 = m(H|G) + m(L|G)$ so that demand can always be met. Independently of his beliefs about the demand state (and the behaviour of other consumers), a consumer will optimally purchase the good (in either period 1 or 2) if and only if the price is lower than his willingness to pay. Clearly, the monopolist will optimally extract all of the surplus from one of the two consumer types. Hence, we can confine attention to two uniform prices, $p = 1$ and $p = v(L)$. The induced probabilities of serving consumers are given by

[17] Note that $P(H|B)$ cancels out so that $P(H|B)$ and $P(H|G)$ are not uniquely determined.

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$$R(H|\sigma) = 1 \quad \text{and} \quad R(L|\sigma) = \begin{cases} 0 & \text{if } p = 1, \\ 1 & \text{if } p = v(\text{L}) \end{cases} \tag{5}$$

for $\sigma = G, B$. These induced probabilities in conjunction with the prices $P(\theta|\sigma) = p$ constitute a mechanism of the class defined above. In particular, the (interim) incentive constraint of the high consumer type is binding, as are the (*ex post*) individual rationality constraints of the low type whenever he obtains the good with positive probability (i.e., $p = v(L)$). Note that the high uniform price, $p = 1$, implements the optimal mechanism if (1) holds, but (2) does not. Similarly, the low uniform price, $p = v(L)$, implements the optimum if (2) holds, but (1) does not. The expected profits can then be obtained by inserting the probabilities (5) into (3).

The profit-maximising uniform price is $p = v(L)$ if

$$\rho\{[m(H|G) + m(L|G)]v(L) - m(H|G)\} + (1-\rho)\{[m(H|B) + m(L|B)]v(L) - m(H|B)\} > 0, \tag{6}$$

and $p = 1$ if the reverse inequality holds. Observe that the first term on the l.h.s. is negative if (1) holds, while the second term is positive if (2) holds.

3.2.3. *Introductory offers*

Next, we consider introductory offers, where $p_1 < p_2$. Independently of his beliefs, each consumer has a dominant strategy when facing an increasing price path, namely to demand the good at the low price in period 1, provided the price is not higher than his willingness to pay. If the consumer is rationed at the low price, his dominant strategy is to demand the good at the high price in period 2, provided again this price is less than his valuation. Clearly, the monopolist has no incentive to ration consumers at the high price. Without loss of generality, she may thus set total capacity $k = m(H|G) + m(L|G)$ so as to always meet demand in the second period. In each period, the monopolist optimally extracts all of the surplus of some consumer type. Under introductory offers, the monopolist will therefore set prices $p_1 = v(L)$ and $p_2 = 1$. We can thus restrict attention to the following family of introductory offer policies, $(v(L), 1, k_1, m(H|G) + m(L|G))$, which is parameterised by k_1. We denote these policies by $IO(k_1)$. Without loss of generality, we assume that $k_1 \leq m(H|G) + m(L|G)$. Expected profits are then given by

$$\begin{aligned} \pi^{IO}(k_1) =& (1-\rho)\Bigg(\{v(L)\min[k_1, m(H|B) + m(L|B)]\} \\ &+ 1\left\{\frac{\max[0, m(H|B) + m(L|B) - k_1]}{m(H|B) + m(L|B)}\right\}m(H|B)\Bigg) \\ &+ \rho\left\{v(L)k_1 + 1\left[\frac{m(H|G) + m(L|G) - k_1}{m(H|G) + m(L|G)}\right]m(H|G)\right\}. \end{aligned}$$

Since this expression is piecewise linear in k_1, the unique candidate for an optimal introductory offer policy is $IO[m(H|B) + m(L|B)]$.[18] That is, the monopolist optimally sets the first-period capacity k_1 so as to just serve all demand at the low price $p_1 = v(L)$

[18] The boundary points $k_1 = 0$ and $k_1 = m(H|G) + m(L|G)$ effectively implement a uniform price of 1 and $v(L)$, respectively.

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when demand is in the bad state. In the good demand state, on the other hand, the monopolist sells $m(H|B) + m(L|B)$ units at the low first-period price and serves all rationed high valuation consumers at the high second-period price. The induced probabilities of serving consumers are thus given by

$$R(H|\sigma) = 1 \quad \text{for} \quad \sigma = G, B; \quad R(L|G) = \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}; \quad \text{and} \quad R(L|B) = 1. \tag{7}$$

These induced probabilities in conjunction with the induced prices $P(L|\sigma) = v(L)$ for $\sigma = G,B$, $P(H|B) = v(L)$, and

$$P(H|G) = \left[\frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right] v(L) + \left[1 - \frac{m(H|B) + m(L|B)}{m(H|G) + m(L|G)}\right] 1$$

constitute a mechanism of the class defined above. In particular, the (interim) incentive constraint of the high consumer type holds with equality, as do the (*ex post*) individual rationality constraints of the low type. The expected profits can be obtained by inserting the probabilities (7) into (3). As can be seen from (7), $IO[m(H|B) + m(L|B)]$ never implements the optimal mechanism.

Comparing the optimal introductory offer policy with the low uniform price $v(L)$, we find that the induced probabilities $R(\theta|\sigma)$ differ only for the low type in the good demand state: under the low uniform price, $R(L|G) = 1$, while under the optimal introductory offer policy, $R(L|G) < 1$.[19] Hence, the introductory offer policy is more profitable than the low uniform price if and only if $R(L|G) = 0$ in the optimal mechanism, i.e., the optimal state-contingent price in the good demand state is 1. That is, $\pi^{IO}[m(H|B) + m(L|B)] > \pi^{U}[v(L)]$ if and only if (1) holds.

Comparing introductory offers with the high uniform price of 1, we find that the induced probabilities differ only for the low type: while low-valuation consumers never purchase the good under the high uniform price, they obtain the good with positive probability in both demand states when the monopolist uses the optimal introductory offer policy. Inserting the difference in the expected probabilities into the expression for the monopolist's expected revenue, (4), we obtain that $\pi^{IO}[m(H|B) + m(L|B)] > \pi^{U}(1)$ if and only if

$$\rho\left[v(L) - \frac{m(H|G)}{m(H|G) + m(L|G)}\right] + (1-\rho)\left[v(L) - \frac{m(H|B)}{m(H|B) + m(L|B)}\right] > 0. \tag{8}$$

We thus obtain the following Proposition.

Proposition 2. *The introductory offer policy* $IO[m(H|B) + m(L|B)]$ *maximises profits among the set of selling policies with* $p_1 \leq p_2$ *if and only if conditions* (1) *and* (8) *are satisfied.*

Hence, if the monopolist were not able to commit *ex ante* to total capacity k, introductory offers would be the optimal selling policy if and only if conditions (1) and (8)

[19] If total demand does not expand in the good demand state, i.e., $m(H|G) + m(L|G) = m(H|B) + m(L|B)$, we have $R(L|G) = 1$ in (7) and the optimal introductory offer strategy is degenerate in that all units are always sold at the low price $v(L)$.

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hold. Observe that if condition (1) holds, the first term on the l.h.s. of (8) is negative. Therefore, for both conditions to hold simultaneously, the second term on the l.h.s. of (8) must be positive, and so condition (2) must hold. That is, for introductory offers to dominate uniform pricing it is necessary (but not sufficient) that the optimal state-contingent price is the low price $v(L)$ in the bad demand state and the high price 1 in the good demand state.

3.2.4. *Clearance sales*

We now consider clearance sales policies, where $p_1 > p_2$. Clearly, the monopolist has no incentive to ration demand at the high price. Without loss of generality, we can therefore set $k_1 = k$. For consumers to be willing to purchase the good in the first period, there must exist a positive probability that consumers are rationed in the second period. Since consumers cannot condition their purchasing decision on the state of the world, but only on their own valuation, any optimal clearance sales policy must have the property that all high-valuation consumers demand the good in the first period, while all low types demand the good in the second period (and are rationed with positive probability). Hence, it is sufficient to consider the family of clearance sales policies (parameterised by capacity k), $(\widehat{p}_1(k), v(L), k, k)$, where $\widehat{p}_1(k)$ is set so as to make high type consumers just indifferent between demanding the good in the first period at price $\widehat{p}_1(k)$, and postponing the purchase (so as to demand the good in the second period at price $v(L)$). We denote these policies by *CS(k)*. For $k \geq m(H|G)$, the indifference condition can be written as

$$1 - \widehat{p}_1(k) = [1 - v(L)]\left\{Q(G|H)\left[\frac{k - m(H|G)}{m(L|G)}\right] + [1 - Q(G|H)]\min\left[1, \frac{k - m(H|B)}{m(L|B)}\right]\right\}, \tag{9}$$

where $\min\{[k - m(H|\sigma)]/m(L|\sigma), 1\}$ is the probability of obtaining the good at the low price in demand state σ. (For $k < m(H|G)$, rationing occurs even at the high price. This case is considered in the proof of Lemma 1.) Since $\widehat{p}_1(k)$ is piecewise linear in k, there are (at most) two potentially optimal capacity levels: $k = m(H|G)$ and $k = m(H|B) + m(L|B)$.

It will prove useful to distinguish between two demand regimes.

- *Weak Demand Shifts.* In this case, the rightward shift of the demand curve is sufficiently small in the sense that the number of high type consumers in the good state is less than the total number of high and low types in the bad state, i.e., $m(H|G) < m(L|B) + m(H|B)$.
- *Strong Demand Shifts.* In this case, the rightward shift of the demand curve is sufficiently large in the sense that the number of high type consumers in the good state is greater than the total number of high and low types in the bad state, i.e., $m(H|G) \geq m(L|B) + m(H|B)$.

The set of potentially optimal clearance sales policies depends on the demand regime. As the following lemma shows, we have to consider only a single clearance sales policy under strong demand shifts and two clearance sales policies under weak demand shifts.

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LEMMA 1. *Under strong demand shifts, the only potentially optimal clearance sales policy is* $CS[m(H|G)]$. *Under weak demand shifts, the only potentially optimal clearance sales policies are* $CS[m(H|G)]$ *and* $CS[m(H|B) + m(L|B)]$.

Proof. See Appendix.

Suppose first that demand shifts are strong, and so $m(H|G) \geq m(H|B) + m(L|B)$. We first consider the clearance sales policy $CS[m(H|G)]$, where, from (9), the first-period price $\widehat{p}_1[m(H|G)]$ is given by

$$\widehat{p}_1 = \frac{\rho m(H|G) + (1-\rho)m(H|B)v(L)}{\rho m(H|G) + (1-\rho)m(H|B)}.$$

That is, the optimal first-period price is a weighted average of $p = 1$ and $p = v(L)$, where the weight on the higher price is $Q(G|H)$, the probability that demand is in the good state, conditional on drawing a high valuation. Capacity $k = m(H|G)$ is such that, in both demand states, high type consumers are not rationed in the first period. In contrast, low type consumers, who demand the good in the second period, are rationed with probability 1 in the good demand state, while they are not rationed in the bad demand state. The induced probabilities of serving consumers are thus given by

$$R(H|\sigma) = 1 \quad \text{for } \sigma = G, B;\ R(L|G) = 0; \quad \text{and} \quad R(L|B) = 1. \tag{10}$$

The expected profits can be obtained by inserting the probabilities (10) into (3). Observe that the optimal clearance sales policy implies the same probabilities as the optimal mechanism if and only if (1) and (2) hold. It follows that the clearance sales policy is revenue equivalent to the (nonfeasible) optimal state-contingent selling policy, where the monopolist charges the high (uniform) price $p = 1$ in the good demand state and the low price $p = v(L)$ in the bad demand state. Note that the revenue equivalence does not necessarily imply the same induced prices for the high type consumers: in the optimal mechanism, $P(H|G)$ and $P(H|B)$ are not uniquely determined. Indeed, $P(H|G) = P(H|B) = \widehat{p}_1[m(H|G)]$ under clearance sales, while $P(H|G) = 1$ and $P(H|B) = v(L)$ under the optimal state-contingent pricing policy.[20]

If conditions (1) and (2) hold, the clearance sales policy $CS[m(H|G)]$ is thus revenue equivalent to the optimal state-contingent selling policy, while uniform pricing and introductory offers lead to lower profits. Moreover, recall that introductory offers are more profitable than uniform pricing *only if* these two conditions are satisfied. Hence, introductory offers are never optimal: they are either dominated by uniform pricing or by clearance sales.

Above, we have shown that the clearance sales policy $CS[m(H|G)]$ is optimal amongst all feasible selling policies when conditions (1) and (2) hold. Which policy is optimal when one of the two conditions is not satisfied? If (1) does not hold,

[20] The clearance sales strategy leads to lower profits than the optimal state-contingent pricing policy when demand is in the good state since the high-valuation consumers obtain the good at price $\widehat{p}_1[m(H|G)] < 1$. The loss in profit is thus equal to $\{1 - \widehat{p}_1[m(H|G)]\}m(H|G)$. In expected terms, this loss is exactly offset by the gain in the bad demand state, which amounts to $\{\widehat{p}_1[m(H|G)] - v(L)\}m(H|B)$.

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then the optimal state-contingent price in the good demand state is $p = v(L)$. In this case, the uniform price $p = v(L)$ dominates the state-contingent policy ($p(G) = 1$, $p(B) = v(L)$): the uniform price yields the same revenues in the bad demand state and higher revenues in the good state. Hence, if (1) does not hold, the clearance sales policy is dominated by the uniform price $p = v(L)$. Similarly, if (2) is not satisfied, then the optimal state-contingent price in the bad demand state is $p = 1$. In this case, the uniform price $p = 1$ dominates the state-contingent policy ($p(G) = 1$, $p(B) = v(L)$) and, by revenue equivalence, the clearance sales policy: the uniform price yields the same revenues as the state-contingent policy in the good demand state and higher revenues in the bad state. We can summarise our results as follows.

LEMMA 2. *Under strong demand shifts, introductory offers are never optimal, and hence the profit-maximising selling policy involves a non-increasing price path,* $p_1 \geq p_2$. *If conditions* (1) *and* (2) *hold, the clearance sales policy* $CS[m(H|G)]$ *is optimal, and hence* $p_1 > p_2$. *Otherwise, uniform pricing is optimal.*

Suppose now that demand shifts are weak and so $m(H|G) < m(H|B) + m(L|B)$. In this case, there are two potentially optimal clearance sales policies, $CS[m(H|B) + m(L|B)]$ and $CS[m(H|G)]$.

As we show in the Appendix, whenever introductory offers dominate uniform pricing, the best introductory offer policy $IO[m(H|B) + m(L|B)]$ is dominated by the clearance sales policy $CS[m(H|B) + m(L|B)]$. The two policies differ only in $R(L|G)$, the probability of serving the low type in the good demand state. Since this probability is smaller under the clearance sales policy, this policy performs better whenever the optimal-state contingent price in the good demand state is 1, which is a necessary condition for introductory offers to dominate uniform pricing. Comparing clearance sales with uniform pricing, it is straightforward to show that $CS[m(H|B) + m(L|B)]$ dominates the low uniform price if (1) holds, while $CS[m(H|G)]$ dominates the high uniform price if (2) holds. While neither $CS[m(H|B) + m(L|B)]$ nor $CS[m(H|G)]$ implement the optimal mechanism under weak demand shifts, clearance sales are optimal if and only if the optimal state-contingent price is $v(L)$ in the bad demand state and 1 in the good demand state; otherwise, uniform pricing is optimal.

LEMMA 3. *Under weak demand shifts, introductory offers are never optimal, and so the profit-maximising selling policy involves a non-increasing price path,* $p_1 \geq p_2$. *If conditions* (1) *and* (2) *hold, clearance sales are optimal, namely either* $CS[m(H|B) + m(L|B)]$ *or* $CS[m(H|G)]$, *and hence* $p_1 > p_2$. *Otherwise, uniform pricing is optimal.*

Proof. See Appendix.

3.2.5. *The main result*

In this article, we have shown that clearance sales may be the optimal selling policy. Perhaps more surprisingly, introductory offers are never optimal in our model. For an introductory offer policy to be more profitable than uniform pricing, there has to be a

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tension between the good and the bad demand state: the *ex post* optimal (uniform) price in the good state has to be higher than the one in the bad state, i.e., $m(H|G) > [m(H|G) + m(L|G)]v(L)$ and $m(H|B) < [m(H|B) + m(L|B)]v(L)$, which are conditions (1) and (2), respectively. However, it is exactly this tension that makes also the clearance sales policy more attractive than uniform pricing. As shown above, in this case, there exists a clearance sales policy which dominates introductory offers as it involves a smaller quantity in the good demand state (and the same quantity in the bad demand state). Combining Lemmas 2 and 3, yields our main result:

PROPOSITION 3. *If conditions* (1) *and* (2) *hold, clearance sales are optimal, and hence* $p_1 > p_2$. *Otherwise, uniform pricing is optimal. Hence, introductory offers are never optimal, and the profit-maximising selling policy involves a non-increasing price path,* $p_1 \geq p_2$.

Note that the clearance sales policy $CS[m(H|G)]$ induces intertemporal dispersion of prices (at which trade occurs) only when demand is in the bad state: in the good demand state, all $m(H|G)$ units are sold at the high first-period price. In contrast, when demand shifts are weak, the clearance sales policy $CS[m(H|B) + m(L|B)]$ employs a larger capacity and induces intertemporal price dispersion in *both* demand states: even in the good demand state, some units are sold at the low second-period price.

4. Discussion

In this Section, we briefly discuss some key assumptions and comment on modifications and extensions.[21]

Capacity Commitment. A clearance sales policy requires a commitment to total capacity before demand uncertainty is resolved.[22] As we have discussed in the introduction, this is an essential feature of the economic environments in which clearance sales are observed. By contrast, uniform pricing does not require any capacity commitment, while an introductory offer policy requires a commitment to first-period capacity. Hence, clearance sales should be less commonly observed in those industries where firms can flexibly adjust capacity.

Price Commitment. In our model with two consumer types and no discounting, our results remain unchanged if the monopolist cannot commit *ex ante* to the second-period price. Under the best clearance sales policy, all high types purchase the good in the first period, and so the ex-post optimal price in the second period is the valuation of the low type. That is, there exists a consumer equilibrium in which the monopolist has no incentive to deviate from the 'announced' price path.[23] A selling policy that

[21] Further discussion of extensions, re-interpretations, and key assumptions is provided in Nocke and Peitz (2004).

[22] If capacity were costly and the unit cost of capacity larger than the valuation of the low consumer type, then the monopolist would have no incentive to increase capacity in the second period even though (low-valuation) consumers are rationed.

[23] With a continuum of consumer types, one would need some unresolved demand uncertainty in the second period to ensure that the monopolist has no *ex post* incentive to raise the second-period price (i.e., to ensure that there is rationing with positive probability in the second period).

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yields the same profit as uniform pricing but does not rely on price commitment involves a price exceeding the high type's valuation in the first period, and the best uniform price in the second period.

Capacity Costs. We have assumed that the monopolist faces zero costs of capacity. Would introductory offers still be dominated if we allowed for positive capacity costs? In our analysis, we have shown that whenever an introductory offer policy performs better than uniform pricing, it is dominated by some clearance sales policy. Consider the generalisation to positive and constant marginal costs of capacity, c_k. As the monopolist changes her policy from a high uniform price to clearance sales to introductory offers to a low uniform price, she has to increase total capacity with each change. This means that with an increase of the capacity cost c_k the condition which ensures that introductory offers dominate the low uniform price becomes less strict. However, the same happens to the condition that clearance sales dominate introductory offers. It can be shown that introductory offers dominate the low uniform price if

$$\rho\{m(H|G) - v(L)[m(L|G) + m(H|G)]\} + c_k m(L|G) > 0.$$

Exactly the same condition implies, however, that clearance sales dominate introductory offers. Hence, the optimal selling policy may still involve clearance sales, but not introductory offers.

Consumer Learning. Here, we have assumed that each consumer learns only his own valuation before making his purchasing decision. In an earlier discussion paper, Nocke and Peitz (2004), we also analyse an alternative information structure where consumers learn not only their own valuations but also the state of demand. While consumers' information about the state of demand is irrelevant for firm and consumer behaviour under uniform pricing and introductory offers, it does affect consumers' behaviour under a decreasing price path. In particular, when consumers learn the state of demand prior to period 1, high-valuation consumers may decide to purchase the good at the high first-period price when demand is in the good state, but at the low second-period price when demand is in the bad state. Nevertheless, all results carry over to the alternative information structure: clearance sales are optimal if and only if the optimal state-contingent price is high in the good demand state and low in the bad demand state; otherwise, uniform pricing is optimal. Under strong demand shifts, the best clearance sales policy implements the optimal mechanism whenever it dominates uniform pricing.

Discounting. In our analysis, we have assumed that there is no discounting which allows us to focus on the roles of rationing and demand uncertainty for the logic underlying clearance sales. Further, we believe this describes the cases of ticket sales and vacation packages very well: those consumers who purchase their ticket or vacation package early cannot consume earlier than those who make the purchase later. But even in the case of seasonal goods, this assumption is not a bad approximation: for example, sales of winter clothing often start well before the worst

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of the cold and snow arrives. Nevertheless it is interesting to ask whether the logic underlying clearance sales relies on our 'no discounting' assumption. The answer is 'no'.

Suppose the monopolist and consumers share a common discount factor δ. Then, it is straightforward to verify that – under demand certainty – uniform pricing continues to be the optimal selling policy.[24] Under demand uncertainty, there generically exists a threshold discount factor $\overline{\delta} < 1$ such that for $\delta \in (\overline{\delta},1)$, the optimal selling policy involves either uniform pricing or clearance sales.[25]

The Number of Selling Periods. Does the monopolist have an incentive to sell over more than two periods? In our simple model with two consumer types, two demand states and consumers learning only their own valuations, the answer is 'no'.[26] To gain some intuition, consider, for instance, the case where the optimal selling policy over two periods involves clearance sales. Suppose the monopolist could sell in period 3 as well. Clearly, the monopolist has no incentive to charge a price below $v(L)$ in period 3; but if she charges a higher price, no consumer would purchase the good in period 3. Suppose now the monopolist could sell in period 0 as well. If the monopolist charges a price that is higher than the price in period 1, no consumer would purchase the good; if she charges a lower price, her profit would fall as some high-valuation consumers would now purchase the good at the lower period-0 price.

The Number of Demand States. Our model can be extended to more than two demand states. The optimal selling policy still involves at most two different prices at which trade occurs with positive probability. Even when demand shifts are 'strong', however, clearance sales do not implement the optimal mechanism. Nevertheless, the logic of clearance sales continues to hold and, as we have been able to show for three demand states, they may still constitute the optimal selling policy.

The Number of Consumer Types. Does the (potential) optimality of clearance sales hinge on our assumption of two consumer types? As we will now show for the case of a continuum of consumer types, the answer is 'no'. Let $F(\cdot|\sigma)$ denote the cumulative distribution function of valuation v amongst consumers in state σ, and M the mass of consumers. (We do not impose any restriction on the correlation across demand states of an *individual* consumer's valuation.) Assume that $F(\cdot|\sigma)$ is such that, in each state σ, the state-contingent marginal revenue curve is downward-sloping. This implies

[24] This result should not be too surprising in light of Stokey's (1979) insight that a monopolist does *not* want to intertemporally price discriminate if all consumers share the same rate of time preference.

[25] The same is true for low discount factors: there exists a threshold $\underline{\delta} > 0$ such that for $\delta \in (0,\underline{\delta})$, either uniform pricing or clearance sales are optimal, while introductory offers are strictly dominated by the low uniform price $v(L)$. However, there exist parameters for which introductory offers can be optimal (in particular, this possibility arises when there is very little demand uncertainty, and the probability of the good demand state is close to zero, i.e., for ρ positive but very small).

[26] Under the alternative information structure, where each consumer learns not only his own valuation but also the state of demand, a hybrid three-price strategy (in addition to clearance sales and uniform pricing) can be optimal under weak demand shifts.

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(Wilson, 1988) that the optimal state-contingent selling policy is a uniform price $p(\sigma)$ given by

$$p(\sigma) = \arg\max_p pM[1 - F(p|\sigma)].$$

To keep this discussion short, we make some (rather strong) simplifying assumptions. First, we assume that $p(G) > \bar{v}(B) \equiv \sup \{v|F(v|B) < 1\}$. This implies that if a consumer learns that his valuation is $v \geq p(G) > \bar{v}(B)$, he will know for sure that the demand state is $\sigma = G$. Second, we assume that the optimal state-contingent output is higher in the good demand state than in the bad demand state, i.e., $F[p(G)|G] < F[p(B)|B]$.

Consider now the clearance sales policy with $p_1 = p(G)$, $p_2 = p(B)$, and $k_1 = k = M\{1 - F[p(G)|G]\} - \varepsilon$, where $\varepsilon > 0$ is small. (We are interested in the limit as $\varepsilon \to 0$.) In the good demand state, each consumer with valuation $v \geq p(G)$ knows for sure that the demand state is $\sigma = G$, and will purchase the good in the first period at price $p(G)$, as he would be rationed with probability one if he were to delay his purchasing decision. (In the limit as ε becomes small, the probability of rationing in the first period goes to zero.) In the bad demand state, each consumer with valuation $v \geq p(B)$ will purchase the good in the second period at price $p(B)$. Hence, the clearance sales policy allows the monopolist to obtain the same profit as with the optimal state-contingent prices.

In contrast, since $p(G) \neq p(B)$, a uniform pricing policy does obviously worse. Further, it is straightforward to show that an introductory offer policy is not optimal. Intuitively, the (low) first-period price p_1 will be less than $p(G)$, which implies that some consumers with valuations less than $p(G)$ will receive the good in the good demand state, whereas under the optimal state-contingent selling policy only consumers with valuations of at least $p(G)$ receive the good when $\sigma = G$.

5. Conclusion

Motivated by the prevalence of clearance sales for season goods, theatre and concert tickets, and holiday packages, we have developed a simple model of monopoly pricing and capacity choice under demand uncertainty. While much of the existing literature has established that uniform pricing is optimal in a large class of economic environments, we have shown that the optimal selling policy may instead involve clearance sales if the monopolist faces (aggregate) demand uncertainty and can commit to selling a limited quantity before learning the state of demand.

In our model, uniform pricing is optimal when demand is certain. Under demand uncertainty, however, clearance sales are the optimal selling policy if and only if the optimal state-contingent price is high in the good state and low in the bad state. Otherwise, uniform pricing is optimal. Interestingly, whenever introductory offers perform better than uniform pricing, they are dominated by clearance sales.

While a profit comparison of the different selling policies seems to be rather complex, we have shown (using a mechanism design approach) that it can be reduced to a comparison of the induced probabilities of serving the two consumer

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types in the two demand states. Suppose the optimal state-contingent price is high in the good demand state and low in the bad state. In this case, the monopolist would optimally like to serve high-valuation consumers with probability one, but low-valuation consumers only in the bad demand state. A high uniform price thus leads to too little trade because, in the bad demand state, low valuation consumers are never served. A low uniform price leads to too much trade because, in the good demand state, low valuation consumers are always served. The best introductory offer policy may dominate both uniform prices as the induced probability of serving low types is less than one (but positive) in the good demand state and one in the bad demand state. However, clearance sales perform even better since they induce a smaller probability of serving low types in the good demand state. In fact, if demand shifts are 'strong' this probability is zero, and so clearance sales implement the optimal state-contingent prices.

Appendix

Proof of Lemma 1. As pointed out in the main text, any optimal clearance sales policy must be such that all high types are just willing to demand the good at the high price. Hence, any optimal clearance sales policy is of the form $(\widehat{p}_1(k), v(L), k)$, where $\widehat{p}_1(k)$ is chosen so as to make high type consumers just indifferent between purchasing at $\widehat{p}_1(k)$ and delaying the purchase.

Step 1. Suppose $k \in [m(H|B), m(H|G)]$. In this case, rationing occurs even at the high first-period price (when demand is in the good state). If $k \leq \min[m(H|G), m(H|B) + m(L|B)]$, a high type consumer is indifferent between purchasing at $\widehat{p}_1(k)$ and postponing his purchase if

$$\left[Q(G|H)\frac{k}{m(H|G)} + 1 - Q(G|H)\right]\left[1 - \widehat{p}_1(k)\right] = [1 - Q(G|H)]\left[\frac{k - m(H|B)}{m(L|B)}\right][1 - v(L)],$$

and so

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)]\left[\dfrac{k - m(H|B)}{m(L|B)}\right]}{1 - \left[\dfrac{m(H|G) - k}{m(H|G)}\right]Q(G|H)}[1 - v(L)].$$

The monopolist's expected profit is then

$$\pi^{CS}(k) = \rho\widehat{p}_1(k)k + (1 - \rho)\{\widehat{p}_1(k)m(H|B) + v(L)[k - m(H|B)]\},$$

which is non-linear in k.

Similarly, if $k \in \{\min[m(H|G), m(H|B) + m(L|B)], m(H|G)\}$, the first-period price is equal to

$$\widehat{p}_1(k) = 1 - \frac{[1 - Q(G|H)]}{1 - \left[\dfrac{m(H|G) - k}{m(H|G)}\right]Q(G|H)}[1 - v(L)],$$

and the expected profit is given by

$$\pi^{CS}(k) = \rho\widehat{p}_1(k)k + (1 - \rho)[\widehat{p}_1(k)m(H|B) + v(L)m(L|B)],$$

which again is non-linear in k.

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Let

$$\overline{p}_1(k) \equiv 1 - [1 - Q(G|H)] \max\left[\frac{k - m(H|B)}{m(L|B)}, 1\right][1 - v(L)],$$

and note that $\overline{p}_1(k) > \widehat{p}_1(k)$ for all $k < m(H|G)$, and $\overline{p}_1[m(H|G)] = \widehat{p}_1[m(H|G)]$. Next, let

$$\overline{\pi}^{CS}(k) \equiv \rho \overline{p}_1(k) m(H|G) + (1 - \rho)\{\overline{p}_1(k) m(H|B) + v(L) \min[k - m(H|B)], m(L|B)\},$$

and note that $\overline{\pi}^{CS}(k) > \pi^{CS}(k)$ for all $k < m(H|G)$, and $\overline{\pi}^{CS}[m(H|G)] = \pi^{CS}[m(H|G)]$. Moreover, observe that $\overline{\pi}^{CS}(k)$ is linear in k for $k \leq \min[m(H|G), m(H|B) + m(L|B)]$, and independent of k on $[m(H|B) + m(L|B), m(H|G)]$.

We now claim that $\overline{\pi}^{CS}[m(H|B)]$ is equal to $\pi^U(1)$, the profit from the uniform price $p = 1$. To see this, note that $\overline{p}_1[m(H|B)] = 1$, and

$$\overline{\pi}^{CS}[m(H|B)] = \rho m(H|G) + (1 - \rho) m(H|B) = \pi^U(1).$$

Since $\overline{\pi}^{CS}(k)$ is linear for $k \leq \min[m(H|G), m(H|B) + m(L|B)]$, it follows that an optimal clearance sales policy must have $k \geq \min\{m(H|G), m(H|B) + m(L|B)\}$. Moreover, since $\overline{\pi}^{CS}(k)$ is constant on $[m(H|B) + m(L|B), m(H|G)]$, strictly larger than $\pi^{CS}(k)$ for all $k < m(H|G)$, and $\overline{\pi}^{CS}[m(H|G)] = \pi^{CS}[m(H|G)]$, an optimal clearance sales policy must have $k \geq m(H|G)$. Hence, there cannot be rationing at the high price.

Step 2. Suppose $k \in [m(H|G), m(H|G) + m(L|G)]$. In this case, rationing can only occur at the low price. The indifference condition for high type consumers can now be written as

$$1 - \widehat{p}_1(k) = \left\{Q(G|H)\left[\frac{k - m(H|G)}{m(L|G)}\right] + [1 - Q(G|H)] \min\left[1, \frac{k - m(H|B)}{m(L|B)}\right]\right\}[1 - v(L)].$$

The expected profit is then

$$\begin{aligned}\pi^{CS}(k) &= \rho\{\widehat{p}_1(k) m(H|G) + v(L)[k - m(H|G)]\}\\ &\quad + (1 - \rho)\{\widehat{p}_1(k) m(H|B) + v(L) \min[m(L|B), k - m(H|B)]\},\end{aligned}$$

which is linear in k on $[m(H|G), m(H|B) + m(L|B)]$, provided this interval is non-empty (i.e., when horizontal demand shifts are weak), and on $[\max\{m(H|G), m(H|B) + m(L|B)\}, m(H|G) + m(L|G)]$.

Hence, under strong horizontal demand shifts (where $m(H|G) \geq m(H|B) + m(L|B)$), the unique candidate for an interior optimum is at capacity $k = m(H|G)$. Under weak horizontal demand shifts (where $m(H|G) < m(H|B) + m(L|B)$), there are two candidates: $k = m(H|G)$ and $k = m(H|B) + m(L|B)$. ■

Proof of Lemma 3. First, consider the clearance sales policy $\{\widehat{p}_1[m(H|B) + m(L|B)], v(L), m(H|B) + m(L|B)\}$, where the first-period price is given by

$$\widehat{p}_1[m(H|B) + m(L|B)] = v(L) + [1 - v(L)]Q(G|H)\left[\frac{m(H|G) + m(L|G) - m(H|B) - m(L|B)}{m(L|G)}\right].$$

If the monopolist employs this clearance sales policy, then all high types will purchase the good in the first period, and all low types try to purchase in the second period. The induced probabilities of serving consumers are given by

$$R(H|\sigma) = 1 \text{ for } \sigma = G, B; R(L|G) = \frac{m(H|B) + m(L|B) - m(H|G)}{m(L|G)}; \text{ and } R(L|B) = 1. \tag{11}$$

The expected profits can be obtained by inserting the probabilities (11) into (3).

Observe that the clearance sales policy $CS[m(H|B) + m(L|B)]$ differs from the introductory offer policy $IO[m(H|B) + m(L|B)]$ only in the induced probability of serving low type consumers in the good demand state, $R(L|G)$. Comparing (11) and (7), it is easily checked that this

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probability is lower under the clearance sales policy. Hence, the clearance sales policy dominates the introductory offer policy if and only if $R(L|G) = 0$ in the optimal mechanism, i.e., if and only if (1) is satisfied. However, recall that condition (1) is necessary for introductory offers to dominate the low uniform price. It follows that introductory offers are never optimal: they are either dominated by the clearance sales policy $CS[m(H|B) + m(L|B)]$ or the low uniform price. By the same argument, the clearance sales policy dominates the low uniform pricing policy if and only if condition (1) holds.

Second, let us consider the clearance sales policy $CS[m(H|G)]$, where the first-period price is given by

$$\widehat{p}_1[m(H|G)] = 1 - \frac{[1 - v(L)](1-\rho)m(H|B)}{\rho m(H|G) + (1-\rho)m(H|B)}\left[\frac{m(H|G) - m(H|B)}{m(L|B)}\right].$$

Under weak demand shifts, the induced probabilities of serving consumers are given by

$$R(H|\sigma) = 1 \text{ for } \sigma = G, B; R(L|G) = 0; \text{ and } R(L|B) = \frac{m(H|G) - m(H|B)}{m(L|B)}. \tag{12}$$

The expected profits can be obtained by inserting the probabilities (12) into (3).

Observe that the induced probabilities differ from those of the high uniform price $p = 1$ only in the higher value of $R(L|B)$. Hence, the clearance sales policy $CS[m(H|G)]$ dominates the high uniform price if and only if $R(L|B) = 1$ in the optimal mechanism, i.e., if and only if (2) is satisfied.

We now claim that the optimal selling policy is a clearance sales policy if and only if conditions (1) and (2) hold. The 'if part' of this claim follows directly from above: the low uniform price is dominated by $CS[m(H|B) + m(L|B)]$ if (1) is satisfied, while the high uniform price is dominated by $CS[m(H|G)]$ if (2) holds. What about the 'only if part'? Clearly, if neither condition (1) nor (2) hold, both clearance sales policies are dominated by a uniform pricing policy. On the other hand, if (1) holds, but (2) does not, then the high uniform price is the only selling policy that implements the optimal mechanism. If the reverse holds, then the low uniform price is the only selling policy that implements the optimum. ■

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